ADVANCED AIRCRAFT PERFORMANCE MODELING FOR ATM: ANALYSIS OF BADA MODEL CAPABILITIES

Damir Poles, Angela Nuic, Vincent Mouillet, EUROCONTROL, Brétigny-sur-Orge (France)

Abstract

This paper provides an analysis of the BADA aircraft performance model capabilities and addresses the BADA model ability to provide accurate modeling of aircraft performances over the complete flight envelope for a number of aircraft types and different ways in which an aircraft can be operated during the flight. The focus of the paper is the support of complex aircraft operations by BADA. A short description of the two existing BADA families and their main characteristics is given. The complex flight instructions and operating regimes – economy climb, cruise and descent based on cost index, maximum range cruise, long range cruise, optimum altitude and maximum endurance cruise – identified as key features in support to optimized flight execution are discussed. The optimization procedures and equations in which they derive are presented and the ability of the BADA model to support these flight operations is demonstrated. It is shown that BADA 4 can be successfully used with complex instructions and operating regimes, whereas the use of BADA 3 is limited. Finally, the results of a validation experiment dedicated to BADA thrust models are presented.

Introduction

Today's and tomorrow's Air Traffic Management (ATM) is faced with numerous challenges: capacity increase, safety improvement, diminished environmental impact and cost decrease [1, 2]. Many research and development (R&D) activities are undertaken in order to design and develop a modern ATM system which would respond to these challenges. Various modeling and simulation tools, which range from mathematical models to fast and real-time simulations, are used to develop and evaluate the new systems. Aircraft trajectory simulation and prediction is one of the key functions of these tools. This function requires an Aircraft Performance Model (APM) to provide geometric, kinetic and kinematic aspects of the aircraft behaviour.

Different modeling and simulation tools have different requirements towards an APM. These requirements may be expressed in terms of:

- accuracy of aircraft performance parameters for specific parts of the aircraft flight envelope,
- support of different aircraft operations and flight phases,
- coverage of different aircraft types,
- levels of complexity and computational requirements.

Some applications might require accurate modeling of the aircraft path and 100 % coverage of the aircraft types in operation today, while others request accurate modeling of fuel consumption and aircraft forces for only several aircraft types.

To address these different requirements, several aircraft performance modeling approaches, and consequently several forms of APM, exist. This paper addresses the EUROCONTROL aircraft performance model called BADA (Base of Aircraft Data) and its ability to meet these requirements.

The paper is structured into four sections. The first section proposes a short overview of the BADA model. The second section describes the key differences between the two BADA model families. The third section discusses the complex flight instructions and operating regimes – such as economy climb, cruise and descent based on cost index, maximum range cruise, or maximum endurance cruise – identified as key features in support to optimized flight execution; the optimization procedures and equations in which they derive are presented, and the ability of the BADA APM to support modeling of these flight operations is demonstrated. The fourth section explores the possibility to independently use the thrust model and presents the results of a validation experiment.

BADA Model Overview

BADA is based on a mass-varying, kinetic approach to aircraft performance modeling [3]. It is structured in three parts: Aircraft Performance Model (APM), Airline Procedure Model (ARPM) and Aircraft Characteristics, as depicted in Figure 1.

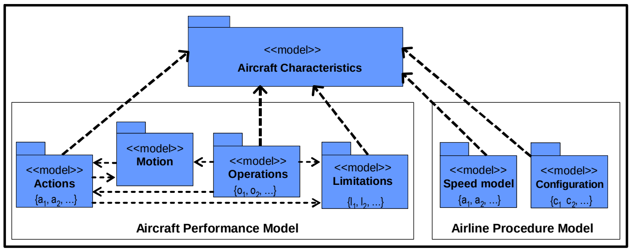

Figure 1. BADA Model Structure

BADA APM is structured into four sub-models: Actions, Motion, Operations and Limitations model.

The **Actions** model represents the forces acting on the aircraft and responsible for its motion. It is divided into three categories – aerodynamic, gravitational and propulsive – including four forces acting on the aircraft (lift, drag, weight and thrust) and fuel consumption which affects the aircraft mass. Therefore, the Actions model consists of the following sub-models: Drag, Lift, Weight, Thrust and Fuel consumption.

The **Motion** model consists of the system of different equations that describes the aircraft motion.

The **Operations** model describes different ways of operating the aircraft that are not part of Actions and Motion model.

The **Limitations** model ensures a realistic aircraft behaviour within certain limits.

A generic **Airline Procedure Model (ARPM)** is proposed in BADA: it provides nominal speeds for the climb, cruise and descent phases, assuming normal aircraft operations.

Each aircraft model in BADA is characterized with a set of coefficients, called **Aircraft Characteristics**, which is used by the APM and ARPM.

More details on sub-models are provided in [3, 4, 5, 6].

BADA APM Families

There are two existing families of BADA: BADA family 3 (BADA 3) which is widely used by the ATM community, and the newly developed BADA family 4 (BADA 4) currently exploited only by EUROCONTROL.

Both BADA families are based on the same modeling approach and have identical structure and components. The primary objective of BADA 3 is to model aircraft behaviour over the normal operations part of the flight envelope, while BADA 4 aims at providing modeling capabilities for the complete aircraft flight envelope.

Keeping this in mind, the main characteristics of the two model families are summarized in this section by applying a set of Key Performance Indicators [1] (KPIs) that are considered most meaningful for assessment of an APM in the context of its use in the ATM.

Capability

The capability of an APM is defined by the performance aspects it is capable of providing, and how they are provided. These performance aspects are classified into four groups that correspond to different aircraft motion features: kinetic, kinematic, operations and limitations [7].

Both BADA 3 and BADA 4 provide the same aspects, while the way they are provided may differ (either directly modeled or computed).

Realism

Realism is defined as the extent to which the model captures the true physical dependences underlying the performance aspects.

While the elaboration of mathematical models for BADA 3 was limited by the low availability of high quality aircraft performance reference data, and a requirement to keep the model algorithms simple because of limited computing capabilities, the development of BADA 4 took advantage of today’s availability and high quality of aircraft performance reference data and significantly improved computing capabilities.

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[1] Defined within the scope of the COURAGE study [7, 8].

The BADA 4 mathematical models were elaborated based on the analysis of the underlying physical laws governing aircraft behaviour and the identification of the physical variables upon which aircraft performance is to be represented [5, 7].

Complexity

Complexity is defined as a qualitative KPI to characterize model specifications. In general, the larger the number of parameters that need to be provided to obtain a specific model, the higher the complexity of the corresponding model specification.

The use of proper physical dependencies and selection of appropriate mathematical models to relate them provide higher accuracy in modeling, but result in an increased number of coefficients in polynomial expressions. As a result, the model specifications are more complex in BADA 4 than in BADA 3: an aircraft model in BADA 4 contains more than 50 coefficients, whereas the equivalent model in BADA 3 contains less than 20 coefficients.

Goodness-of-Fit

Goodness-of-fit measures how well a specific model fits the reference data set from which it has been derived. The reference data set for BADA 3 covers the part of the flight envelope corresponding to nominal operating conditions: it considers climb and descent aircraft operations from low to high operating speeds (as operated by airlines), at weights ranging from minimum to maximum weight, and atmospheric conditions expressed as the temperature deviation (ΔT) from the International Standard Atmosphere (ISA) conditions ranging from ISA+0 to ISA+20. For these conditions, the BADA 3 model demonstrates the ability to predict aircraft performances with a mean root mean square (RMS) error in vertical speed lower than 100 fpm and a fuel flow error less than 5%.

The BADA 4 reference data set comprises the complete flight envelope. For a validation set of 25 aircraft types, the mean error in vertical speed is less than 70 fpm and the fuel flow error well below 5% for the complete aircraft operations envelope. This considers climb and descent aircraft operations from minimum to maximum operating speeds (V_{mo}/M_{mo}), aircraft weights ranging from operational empty weight to maximum take-off weight, and atmospheric conditions from ISA-20 to ISA+30.

Accuracy

Accuracy measures how well a specific model reproduces the validation data set, which is independent of the data used to fit the model.

The validation set used in both BADA families covers as much of the complete flight envelope as possible, which is important for the determination of the domain of validity.

The domain of validity of BADA 3 is the nominal flight envelope, where the error levels in accuracy remain similar to the error levels in goodness-of-fit. The error increases towards the edges of the flight envelope: the model can still be used outside of its domain of validity, but with reduced accuracy.

The domain of validity of BADA 4 is the complete flight envelope and the error levels in accuracy remain similar to the error levels in goodness-of-fit.

Maintainability

Maintainability is defined as the amount of resources required for an APM to be kept up-to-date, and depends on the degree of automation of the model identification process.

Almost the same development process [6] is used for the development of BADA 3 and 4 – only the tools used for identification are different for each family – and the same amount of resources is needed for the generation of new aircraft instances and the maintenance of existing aircraft instances.

Applicability

Applicability defines the area of application of each capability provided by an APM. For each provided performance aspect, the following attributes are considered: coverage (aircraft types), scope (operating regimes[2] and operating configurations[3]) and completeness (flight envelope) [7].

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[2] The operating regime is defined as the strategy adopted to fly within a certain phase of flight. For example, en-route climb at constant calibrated airspeed (CAS) with maximum climb power

The latest revision of BADA 3 provides a high coverage of air traffic, with 318 different aircraft types supported – 111 directly modeled and 207 supported as synonyms [5]. The coverage of aircraft types by BADA 4 is closely related to the availability of high quality aircraft performance reference data: at the moment, this kind of data exists only for major airliners.

The improved accuracy and realism of the underlying aircraft forces in BADA 4, including accurate estimation of drag, thrust and fuel, support modeling of complex operations and operating regimes, such as maximum range cruise or economy cruise. Theoretically, these operating regimes are treatable with the BADA 3 model too. The BADA 3 model realism, accuracy and complexity, however, are limiting factors and the obtained results are not of a good quality.

In addition, BADA 4 models more operating configurations than BADA 3 [3, 4, 7, 8].

Complex Operations Supported by the BADA Model

Management of aircraft operations in terms of optimization of the flight cost has increasing relevance in the context of today's economical and environmental aspects of ATM systems.

This section provides an analysis of optimized flight operations to obtain the equations in which they derive, and demonstrates the ability of the BADA APM to support modeling and simulation of complex operations.

The following complex operations are considered:

- Cost management based on Cost Index (CI) in climb, cruise and descent;
- Cruise management including: maximum range cruise, long range cruise, optimum altitude, holding (maximum endurance cruise).

A sufficient level of realism and accuracy of an APM Actions model, in particular the Fuel consumption model, is a prerequisite for the successful modeling of these functions, and will be the focus of the following analysis.

Cost Management of Aircraft Operations Based on the Cost Index (CI)

The objective of the cost management is minimization of the total flight cost C_t (also called direct operating cost) [9, 10, 11]:

$$\begin{aligned} C_t &= C_{fix} + C_{var} \\ &= C_{fix} + C_F\,\Delta m + C_T\,\Delta t \qquad (1) \\ &= C_{fix} + C_F\,(\Delta m + \Delta t\; C_T / C_F), \end{aligned}$$

where C_{fix}, C_{var}, C_T and C_F are respectively the fixed, variable, time related and fuel related costs, and Δt and Δm are the period of time and the fuel consumed. As C_{fix} and C_F do not change frequently, at least during a flight, the minimization can be done by the optimization of C_T/C_F ratio.

The cost index (CI) is defined as the ratio between time and fuel related costs [9, 10, 11, 12]:

$$CI = \frac{C_T}{C_F}\ \text{[kg/min]}. \qquad (2)$$

The fundamental rationale of the cost index concept is to achieve minimum flight trip cost by means of a trade-off between operating costs per hour and incremental fuel burn.

The range of CI values usually varies from 0 to 99 or 999 [kg/min] in function of the aircraft manufacturer. Extreme values $CI = 0$ ($C_T << C_F$) and $CI = CI_{max}$ ($C_T >> C_F$) represent minimum fuel mode and minimum flight time mode respectively. $CI = 20$ may be interpreted as the cost of 20 kilograms of fuel being equal to the cost of 1 flight minute.

For a given sector and a predefined value of CI[4], minimum flight cost is achieved by adopting an

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defines an operating regime for the en-route phase. Thus, the operating regime for which a performance is provided is specified by a phase of flight together with a control strategy to fly that phase.

[3] The aircraft may be in different operating configurations within an operating regime. The configurations of interest are those related to the devices that influence aircraft performance: high-lift devices configuration, anti-ice on/off, air conditioning setting, landing gear position, speed brakes, etc.

[4] Airlines usually define cost indices per flight leg taking into account the airline cost structure and operating priorities. They are seasonally readjusted to account for recurring fluctuations. The determination of CI varies across airlines and will not be considered in the paper.

operational speed that properly proportions both fuel and time related costs. The CI is entered into the aircraft Flight Management Computer (FMC) which calculates the most economic (ECON) speed for each phase of a specific flight.

The identification of the ECON speed is considered as an advanced function that an APM can provide. The procedure and equations to obtain it are presented here after.

Cost Index Cruise Management

CI cruise management is based on the determination of an optimum cruise speed, called economic Mach number M_{ECON}. M_{ECON} minimizes the total cost of the phase, C_t, for given values of: CI, aircraft weight (W), cruise geopotential pressure altitude (H_p) and atmospheric conditions expressed as the temperature deviation (ΔT) from the ISA conditions – in the sake of simplicity, the effect of wind and pressure deviation are neglected further in the paper.

The minimization of the total cost is reduced to the minimization of the economy cruise cost function (ECCF) [11]:

$$ECCF = \frac{C_{\text{var}}}{C_F \, \Delta r} = \frac{CI + F}{v_{gr}} \quad , \tag{3}$$

where F is the fuel consumption, Δr is the flown distance and v_{gr} is the ground speed. The solution of the minimization of ECCF is the economic Mach number, M_{ECON}.

Finally, the procedure to determine M_{ECON} is the following: for the given CI, W, H_p and ΔT, find $M' = \left\{ M_i \mid i \in N, M_i \in R, 0 < M_i \le M_{mo}, \frac{dECCF}{dM}(M_i) = 0 \right\}$ where M_{mo} is the maximum operating Mach number, and find M_k, $M_k \in M'$, with the minimum ECCF value, $ECCF(M_k) = \min\{ECCF(M_i) \mid M_i \in M'\}$. The result Mach number is M_{ECON}, $M_{ECON} = M_k$.

The expected behaviour of M_{ECON} and CI may be summarized as [9, 10, 11]:

- for given H_p and W, a higher CI implies a higher M_{ECON},;
- for a given CI, a higher H_p implies a higher M_{ECON} and a higher W implies a higher M_{ECON}.

Figures 2 and 3 present M_{ECON} values calculated using the described procedure with BADA 4 for $CI \in \{0, 10, 20, 40, 60\}$, for a medium size twin jet aircraft model under ISA conditions. In Figure 2 M_{ECON} values are calculated for constant $H_p = 32000$ [ft], while in Figure 3 M_{ECON} values are calculated for constant $W = 65000$ [kg].

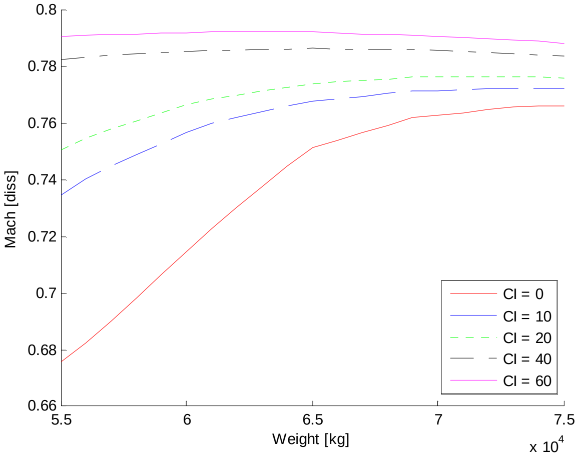

Figure 2. M_{ECON} as $f(W)$, $H_p = 32000$[ft]

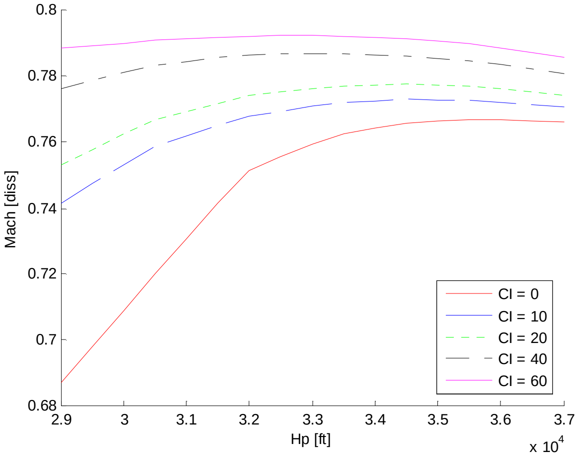

Figure 3. M_{ECON} as $f(H_p)$, $W = 65000$ [kg]

Figures 2 and 3 demonstrate that, using BADA 4, the behaviour of M_{ECON} with respect to W and H_p matches the expected behaviour.

In the case of BADA 3, however, the minimization of ECCF with respect to Mach number (M) does not show the expected behaviour, e.g. for jet aircraft ECCF reaches its minimum for the highest Mach number regardless of the CI value. This is due to the lower level of realism and accuracy of the BADA 3 Actions model, which is therefore not applicable for the different fuel-based optimizations described in this paper, such as cost management based on CI or cruise management.

To better support this conclusion regarding BADA 3, a brief accuracy analysis of BADA 3 and BADA 4 Fuel consumption models in cruise is of interest. For the same aircraft model, fuel consumption values calculated using BADA 3 and 4 are compared to the fuel consumption reference data for two cruise cases:

- constant M, fuel consumption as a function of H_p (22000 – 38000 [ft]);
- constant calibrated airspeed (CAS), fuel consumption as a function of H_p (5000 – 28000 [ft]).

The results for a given $W = 64000$ [kg] and ISA conditions are depicted in Figure 4 and 5.

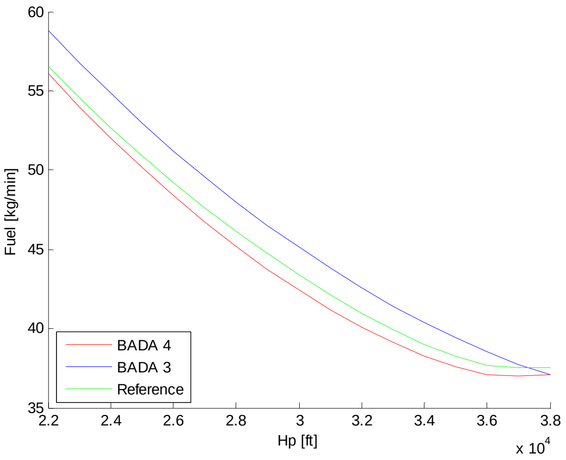

Figure 4. Fuel for Constant *M* and High H_p

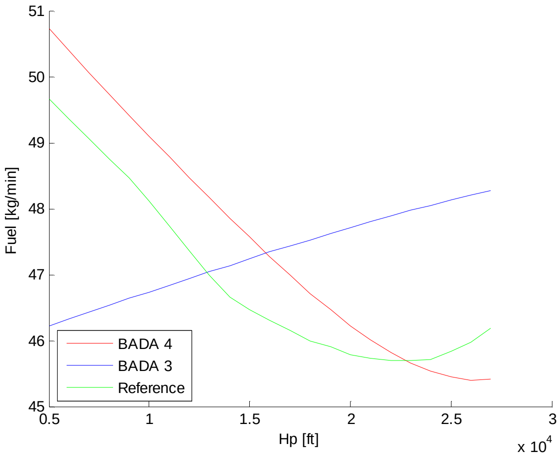

Figure 5. Fuel for Constant *CAS* and Low H_p

Figure 4 focuses on the higher altitudes where both models follow the trend of the reference data, but with a better accuracy for BADA 4. Figure 5 indicates that the cruise fuel consumption model for BADA 3 has significantly lower accuracy results than BADA 4, and demonstrates its inability to follow the trend of reference data on lower altitudes.

Cost Index Climb Management

CI climb management is based on the determination of an optimum climb speed schedule, composed of economic climb speed CAS'_{ECON} (calibrated airspeed) and Mach number M'_{ECON} that form the economic climb speed schedule $250[kt]/CAS'_{ECON}\ /M'_{ECON}$ [5] [10]. This climb speed schedule minimizes the total cost of the climb phase for given values of: CI, aircraft weight, cruise geopotential pressure altitude ($H_{p,CR}$) and atmospheric conditions. The optimization is performed over a specified distance, called range, which includes the climb phase to the given cruise altitude and the economy cruise phase from the top of climb (TOC) to the end of the specified range. M'_{ECON} is assumed to coincide with M_{ECON} from the cruise phase at the given cruise altitude, appropriate aircraft weight and atmospheric conditions.

The extreme values of CI, $CI = 0$ and $CI = CI_{max}$ represent maximum rate of climb (minimization of fuel consumption) and maximum climb speed (minimization of flight time: $CAS'_{ECON} = V_{mo}$, where

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[5] 250 kt CAS below Flight Level 100 (FL100), CAS'_{ECON} above FL100 until transition with M'_{ECON}.

V_{mo} is the maximum operating speed) respectively [9, 11, 12].

The procedure to determine the economic climb speed schedule in BADA is the following:

- definition of three climb segments *(named after their respective speeds* CAS_1, CAS_2, M*)* and one economy cruise segment (M) across a specified range of 500 [nm];
- definition of the total cost function as: $cost = cost(CAS_1) + cost(CAS_2) + cost\_climb(M) + cost\_cruise(M)$;
- determination of CAS_2 and M ($CAS_1 = 250$) and calculation of the total cost;
- minimization of the cost function and determination of CAS'_{ECON} and M'_{ECON}.

The cost function, for a given airspeed in the segment, is defined as: $cost(CAS/M) = CI\ \Delta t + \Delta m$ [kg], where Δt and Δm are the time spent and the fuel consumed in the segment for a given CAS/M.

It is expected that a higher CI causes higher CAS'_{ECON} and M'_{ECON}, a longer climb distance, a later start of cruise segment (TOC) and a shallower flight path [9, 10, 11].

Table 1 presents CAS'_{ECON} and M'_{ECON} values calculated using the above described procedure with BADA 4 for $CI \in \{0, 20, 40, 60, 100\}$, for a medium size twin jet aircraft model, $H_{p,CR} = 33000$ [ft], $W = 75000$ [kg] and ISA conditions.

Table 1. CAS'_{ECON} and M'_{ECON} Climb Results

Cost Index	CAS'_{ECON}	M'_{ECON}
0	311	0.7664
20	327	0.7763
40	339	0.7837
60	349	0.7884
100	350	0.7959

The resulting vertical profiles for different CIs and associated economic climb speed schedules are shown in Figure 6.

Table 1 and Figure 6 demonstrate that, using BADA 4, the behaviour with respect to W and $H_{p,CR}$ of the economic climb schedule and the resulting vertical profile match the expected behaviour.

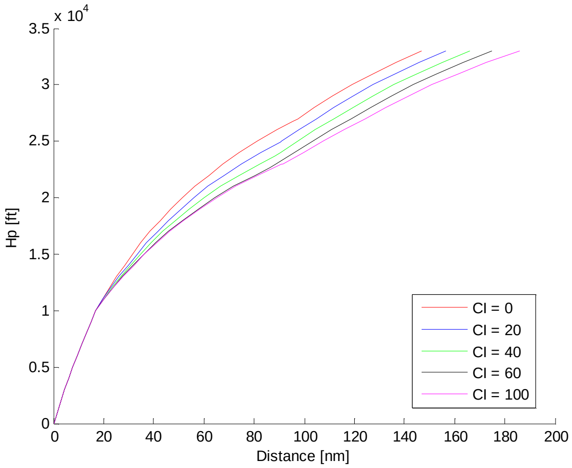

Figure 6. Climb Phase, H_p as $f(r)$

Cost Index Descent Management

CI descent management is based on the determination of an optimum descent speed schedule, composed of economic descent speed CAS''_{ECON} (calibrated airspeed) and Mach number M''_{ECON} that form the economic descent speed schedule $250[kt]/CAS''_{ECON}\ /M''_{ECON}$ [6][10] This descent speed schedule minimizes the total cost of the descent phase for given values of: CI, aircraft weight, cruise geopotential pressure altitude ($H_{p,CR}$) and atmospheric conditions. The optimization is performed over a specified range which includes the economy cruise phase at the given cruise altitude up to the top of descent (TOD) and the descent phase from the TOD to the end of the specified range. M''_{ECON} is assumed to coincide with M_{ECON} from the cruise phase at the given cruise altitude, appropriate aircraft weight and atmospheric conditions.

The extreme values of CI, $CI = 0$ and $CI = CI_{max}$ represent minimum descent speed (minimization of fuel consumption) and maximum descent speed ($CAS''_{ECON} = V_{mo}$, reduction of flight time).

Note that the descent optimization problem is less complex than the climb one: since the aircraft weight at the beginning of the cruise phase is now given, $M_{ECON} = M''_{ECON}$ can here be determined directly from the cruise segment using the economy cruise procedure.

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[6] 250 kt CAS below FL100, CAS''_{ECON} above FL100 until transition with M''_{ECON}.

The procedure to determine the economic descent speed schedule in BADA is the following:

- definition of one economy cruise segment (M) and three descent segments (CAS_1, CAS_2, M) across a specified range of 400 [nm];
- definition of the cost function (the same as for the climb management);
- determination of CAS_2 and M ($C_1 = 250$), calculation of the cost;
- minimization of the cost function and determination of CAS''_{ECON} and M''_{ECON}.

It is expected that a higher CI causes higher CAS''_{ECON} and M''_{ECON}, a later descent (TOD), a shorter descent distance and a steeper descent [9, 10, 11].

Table 2 presents CAS''_{ECON} and M''_{ECON} values calculated using the above described procedure with BADA 4 for $CI \in \{0, 20, 40, 60, 100\}$, for a medium size twin jet aircraft model, $H_{p,CR} = 37000$ [ft], $W = 62000$ [kg] and ISA conditions.

Table 2. CAS''_{ECON} and M''_{ECON} Descent Results

Cost Index	CAS''_{ECON}	M''_{ECON}
0	248	0.7665
20	268	0.7778
40	306	0.7857
60	333	0.7913
100	349	0.7986

The resulting vertical profiles for different CIs and associated economic descent speed schedules are shown in Figure 7.

Table 2 and Figure 7 demonstrate that, using BADA 4, the behaviour with respect to W and $H_{p,CR}$ of the economic descent speed schedule and the resulting vertical profile match the expected behaviour.

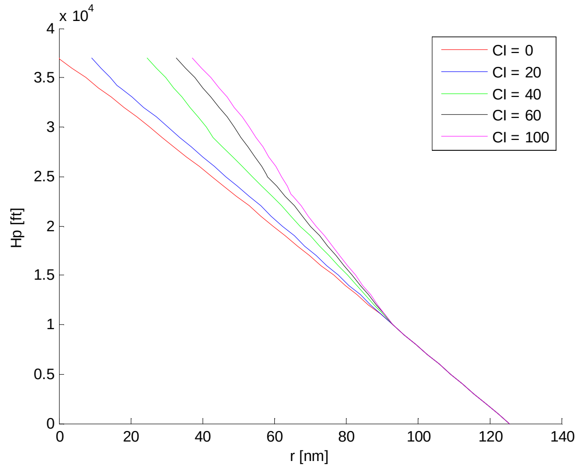


Figure 7. Descent Phase, H_p as $f(r)$

Management of Aircraft Cruise Operations

Besides the optimum flight speeds based on the Cost Index, other flight optimization procedures can be defined according to different optimization criteria: the following sub-sections will present several criteria that are commonly used to optimize the cruise phase of a flight.

Maximum Range Cruise (MRC)

The objective of the MRC is to maximize the flight range for given values of fuel load and atmospheric conditions, usually at constant H_p. The solution is the maximum range Mach number M_{mrc}, which achieves the minimum fuel consumption with respect to distance, or equivalently the maximum distance the aircraft can fly with the given fuel at the given altitude. It is considered as the extreme case of CI cruise management with a CI value equal to zero.

In the BADA MRC optimization procedure, the maximization of flight range at constant H_p is reduced to the maximization of the specific range (SR) with respect to Mach number. The specific range is defined as the distance that can be flown per unit of fuel [10, 11, 13]:

$$SR = -\frac{dr}{dm} = \frac{v_{gr}}{F} \text{ [nm/kg]}. \quad (4)$$

The BADA MRC procedure is the following: for the given W, H_p and ΔT, find $M' = \left\{ M_i \mid i \in N, M_i \in R, 0 < M_i \le M_{mo}, \frac{dSR}{dM}(M_i) = 0 \right\}$ and find M_k, $M_k \in M'$, with the maximum SR value,

$SR(M_k) = \max\{SR(M_i) \mid M_i \in M'\}$. The result is the maximum range Mach number M_{mrc}, $M_{mrc} = M_k$.

It is expected that an increase in H_p causes an increase in M_{mrc}, and a decrease in W causes a decrease in M_{mrc}. During the flight, the aircraft's weight is decreasing, SR is increasing and M_{mrc} is decreasing: to achieve the maximum range, the Mach number shall thus be adjusted to weight changes during the flight [10, 11].

Figures 8 and 9 show the behaviour of M_{mrc} for a medium size twin jet aircraft model using BADA 4. In Figure 8 M_{mrc} is given as a function of W with variations in H_p, while in Figure 9 SR for M_{mrc} is given as a function of W with variations in H_p.

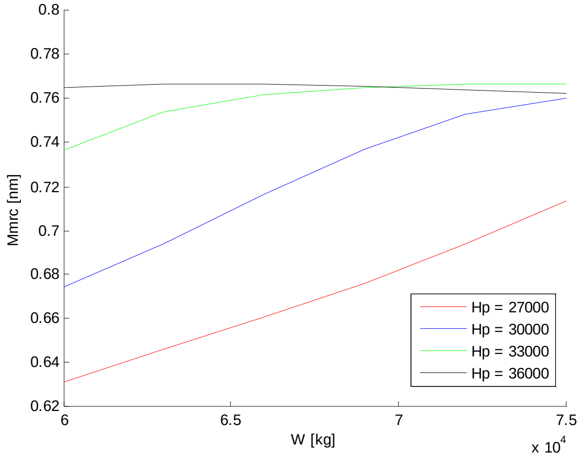


Figure 8. Maximum Range, M_{mrc} as *f(W)*

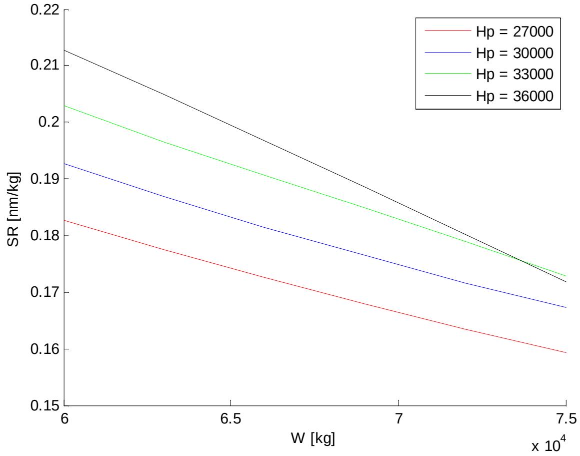


Figure 9. Maximum Range, *SR*, as *f(W)*

Figures 8 and 9 demonstrate that, using BADA 4, the behaviour of SR and M_{mrc} with respect to W and H_p match the expected behaviour.

Long Range Cruise (LRC)

In comparison to the MRC, a slight increase in fuel consumption allows a significant increase in Mach number and a reduction in flight time for the same conditions. SR of the LRC corresponds to 99% of the MRC *SR*, and LRC Mach number M_{lrc} is defined as: $0.99 * SR(M_{mrc}) = SR(M_{lrc}), M_{lrc} > M_{mrc}$ for the given W, H_p and atmospheric conditions [10, 11, 13].

BADA LRC optimization procedure is based on BADA MRC optimization procedure, extended to determine M_{lrc} according to its definition.

Holding, Maximum Endurance Cruise (MEC)

When holding is requested, the knowledge of maximum holding time (endurance) is of most importance for the decision making process. It is defined as the maximization of the time an aircraft can remain airborne with a given amount of fuel [10, 11, 13], i.e. the fuel consumption is minimized with respect to time. As for MRC and LRC, the most important case is the constant H_p case and the minimization of fuel consumption with respect to the Mach number for the given W and atmospheric conditions. The result of the minimization is the holding speed, called holding Mach number M_{mec}, defined as the speed at minimum allowable fuel flow (flame-out) [13]. Since this speed falls into the speed-instability region (near the minimum drag speed and the maximum *L/D* speed), it is usually increased slightly to provide easier aircraft control [11].

The BADA Holding optimization procedure is the following: for the given W, H_p and ΔT, find

$$M' = \left\{ M_i \mid i \in N, M_i \in R, 0 < M_i \le M_{mo}, \frac{dF}{dM}(M_i) = 0 \right\}$$

and find M_k, $M_k \in M'$, with the minimum value of F, $F(M_k) = \min\{F(M_i) \mid M_i \in M'\}$. The result is the maximum endurance Mach number M_{mec}, $M_{mec} = M_k$. At the end of the procedure M_{mec} is slightly increased: $M_{mec} = M_{mec} + \Delta M$.

Optimum Altitude

The optimum altitude $H_{p,opt}$ is defined as the geopotential pressure altitude at which the SR is maximum for given values of: Mach number (M), aircraft weight and atmospheric conditions [10]. This may be considered as an MRC case with constant M [11, 13]. The optimum altitude corresponds to the maximum lift to drag ratio (L/D) or the maximum lift coefficient to drag coefficient (C_L/C_D): the optimum altitude optimization procedure may thus be reduced to the maximization of the lift to drag ratio.

It is expected that [10, 11]:

- as W is decreasing, SR and optimum altitude $H_{p,opt}$ are increasing for a given Mach number M;
- as M is increasing, SR and optimum altitude $H_{p,opt}$ are decreasing for a given aicraft weight W when an aircraft flights at high speed.

The BADA optimum altitude optimization procedure is the following: for the given M and ΔT, find

$$H_p' = \left\{ H_{p,i} \mid i \in N, H_{p,i} \in R, 0 < H_{p,i} \le H_{mo}, \frac{dSR}{dH_p}\left(H_{p,i}\right) = 0 \right\}$$

where H_{mo} is the maximum operating altitude and find $H_{p,k}$, $H_{p,k} \in H_p'$, with the maximum value of SR, $SR(H_{p,k}) = \max\{SR(H_{p,i}) \mid H_{p,j} \in H_p'\}$. The result is optimum altitude $H_{p,opt}$: $H_{p,opt} = H_{p,k}$.

Figure 10 and 11 show results for a medium size twin jet aircraft model in BADA 4. Figure 10 depicts SR (and $H_{p,opt}$) as a function of H_p for a given $W = 70000$ [kg] with respect to different M, while Figure 11 depicts SR (and $H_{p,opt}$) as a function of H_p for a given $M = 0.78$ with respect to different W.

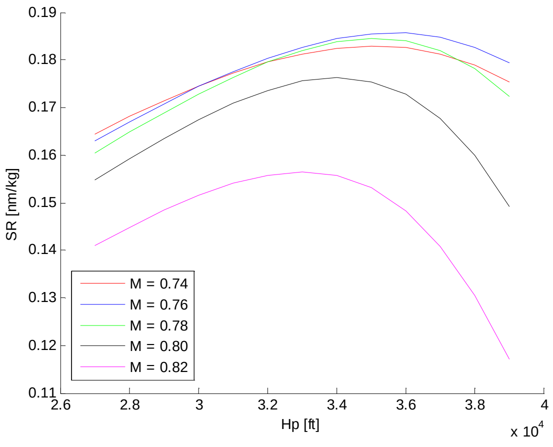


Figure 10. ***SR* as *f(H*$_p$*)*, BADA 4, *W = 70000* [kg]**

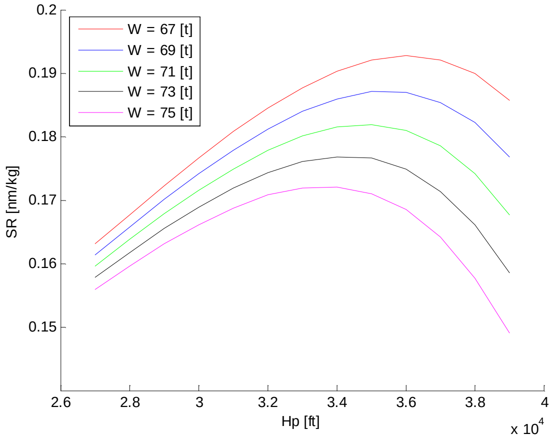


Figure 11. ***SR* as *f(H*$_p$*)*, BADA 4, *M = 0.78***

Figures 10 and 11 demonstrate that, using BADA 4, the behaviour of SR and $H_{p,opt}$ with respect to W and M match the expected behaviour.

Figure 12 depicts SR (and $H_{p,opt}$) as a function of H_p with respect to different W, for the same conditions as Figure 11 – same aircraft, $M = 0.78$ – but this time using BADA 3. It can be seen that the optimum altitude computed with BADA 3 for all values of W is equal to the highest altitude, which is not the expected behaviour. For lower altitudes (i.e. up to 34000 [ft]), however, BADA 3 SR shows a correct behaviour and good accuracy in comparison to BADA 4 SR (Figure 11). The main reason for that is a limitation in the BADA 3 Drag model, which does not take into account the compressibility effect that appears at high altitudes and speeds.

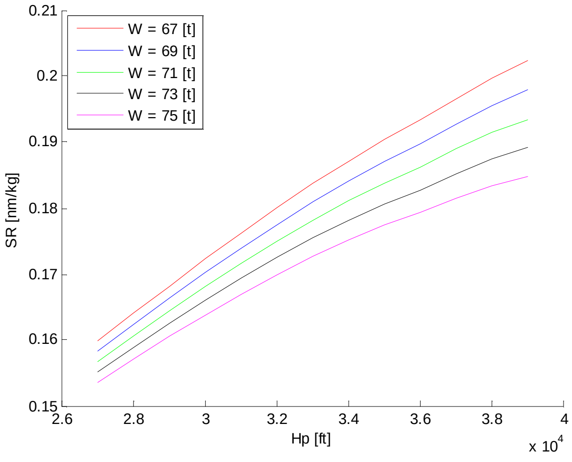


Figure 12. ***SR* as *f(H*$_p$*)*, BADA 3, *M = 0.78***

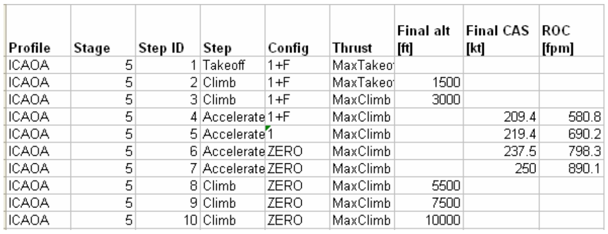
Independent Use of BADA Thrust Model

The independent use of BADA sub-models, such as the Thrust and Fuel consumption models, is solicited by some specific areas of application. These are often related to modeling of environmental aspects of ATM, including aircraft noise and emission modeling where accurate models of engine thrust and fuel consumption play a crucial role. The independent use of BADA sub-models is possible as long as the models meet the levels of accuracy requested by the application.

The accuracy measurement of the thrust model is often difficult, as the original engine data is not easily available[7]. For this reason, different validation experiments can be performed in an attempt to assess the quality of the results. This section presents results of a validation experiment which aims at comparing the BADA Thrust model against a recognized existing model: the Thrust model defined by the Society of SAE AIR (Automotive Engineers Aerospace Information Report) 1845 methodology [14, 15]. This is done by comparing the thrust values along climb profiles calculated for a medium size twin jet aircraft whose performance model (for a specific combination of airframe and engine) is supported by BADA 3, BADA 4 and the Aircraft Noise and Performance database (ANP) [16][8].

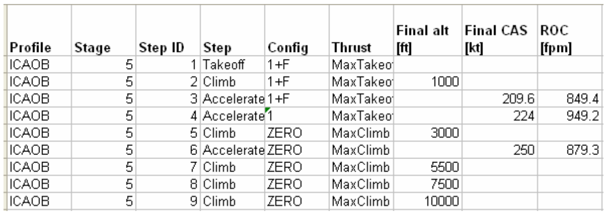
Two different generic climb profiles, referred to as the International Civil Aviation Organization A (ICAO A) and ICAO B and described in [15], are used for the comparison. Each profile is defined with procedural steps that prescribe how the profile is flown (aircraft weight, flight configuration, power setting, forward speed, vertical speed, etc). Details about the ICAO A and ICAO B profiles are depicted in Figure 13 and 14.

Different stages take into account different aircraft's take-off weights (TOW): Stage 1 *TOW = 61204* [kg], Stage 2 *TOW = 63703* [kg], Stage 3 *TOW = 66300* [kg], Stage 4 *TOW = 68201* [kg] and Stage 5 *TOW = 77000* [kg] for both profiles. The stages are defined with different Final CAS and vertical speed (i.e. rate of climb (ROC)) values, while Final altitude values are the same for all stages of a given profile (Figures 13 and 14).



Profile	Stage	Step ID	Step	Config	Thrust	Final alt [ft]	Final CAS [kt]	ROC [fpm]
ICAOA	5	1	Takeoff	1+F	MaxTakeo			
ICAOA	5	2	Climb	1+F	MaxTakeo	1500		
ICAOA	5	3	Climb	1+F	MaxClimb	3000		
ICAOA	5	4	Accelerate	1+F	MaxClimb		209.4	580.8
ICAOA	5	5	Accelerate	1	MaxClimb		219.4	690.2
ICAOA	5	6	Accelerate	ZERO	MaxClimb		237.5	798.3
ICAOA	5	7	Accelerate	ZERO	MaxClimb		250	890.1
ICAOA	5	8	Climb	ZERO	MaxClimb	5500		
ICAOA	5	9	Climb	ZERO	MaxClimb	7500		
ICAOA	5	10	Climb	ZERO	MaxClimb	10000		

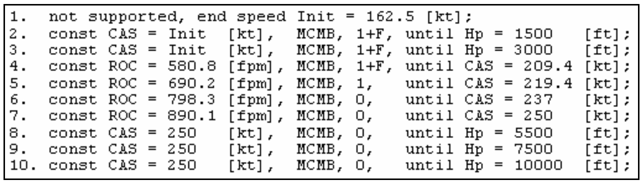
Figure 13. ICAO A Stage 5



Profile	Stage	Step ID	Step	Config	Thrust	Final alt [ft]	Final CAS [kt]	ROC [fpm]
ICAOB	5	1	Takeoff	1+F	MaxTakeo			
ICAOB	5	2	Climb	1+F	MaxTakeo	1000		
ICAOB	5	3	Accelerate	1+F	MaxTakeo		209.6	849.4
ICAOB	5	4	Accelerate	1	MaxTakeo		224	949.2
ICAOB	5	5	Climb	ZERO	MaxClimb	3000		
ICAOB	5	6	Accelerate	ZERO	MaxClimb		250	879.3
ICAOB	5	7	Climb	ZERO	MaxClimb	5500		
ICAOB	5	8	Climb	ZERO	MaxClimb	7500		
ICAOB	5	9	Climb	ZERO	MaxClimb	10000		

Figure 14. ICAO B Stage 5

The climb profiles based on the BADA 3 and 4 APM are calculated using EUROCONTROL's Trajectory Computation Infrastructure (TCI) [17]. This requires ICAO flight procedure steps to be converted into flight intent segments that can be used as inputs to the TCI. Figure 15 presents the TCI input segments corresponding to the ICAO A Stage 5 profile (Figure 13).



```
1.  not supported, end speed Init = 162.5 [kt];
2.  const CAS = Init  [kt],  MCMB, 1+F, until Hp = 1500   [ft];
3.  const CAS = Init  [kt],  MCMB, 1+F, until Hp = 3000   [ft];
4.  const ROC = 580.8 [fpm], MCMB, 1+F, until CAS = 209.4 [kt];
5.  const ROC = 690.2 [fpm], MCMB, 1,   until CAS = 219.4 [kt];
6.  const ROC = 798.3 [fpm], MCMB, 0,   until CAS = 237   [kt];
7.  const ROC = 890.1 [fpm], MCMB, 0,   until CAS = 250   [kt];
8.  const CAS = 250   [kt],  MCMB, 0,   until Hp = 5500   [ft];
9.  const CAS = 250   [kt],  MCMB, 0,   until Hp = 7500   [ft];
10. const CAS = 250   [kt],  MCMB, 0,   until Hp = 10000  [ft];
```

Figure 15. ICAO A Stage 5 in TCI Format

The climb profile based on SAE AIR 1845 methodology and ANP data is calculated using the Integrated Noise Model (INM) tool from the Federal Aviation Administration (FAA) [18].

Figure 16 shows the resulting true airspeed (TAS) in function of H_p for the ICAO A stage 5 profile.

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[7] The identification of BADA Thrust and Drag models is done indirectly using aircraft profile data [6], which do not provide the possibility to measure their respective goodness-of-fit and accuracy.

[8] The ANP database is an online data resource accompanying [14] and [15].

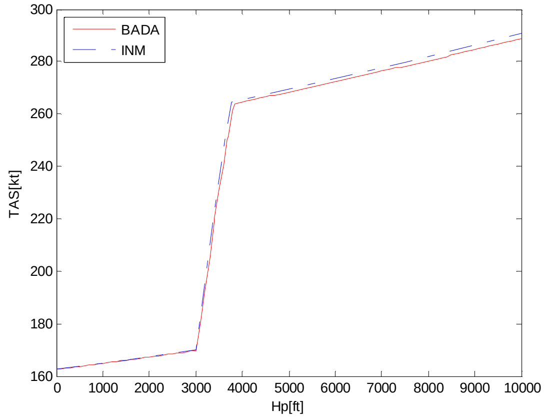

Figure 16. TAS for ICAO A Stage 5

The small discrepancies in TAS are caused by a different calculation of speed in the acceleration steps between the INM model, whose inputs specify an average ROC over the acceleration step, and the TCI, whose inputs specify a constant ROC over the acceleration step. The maximum error in TAS is less than one percent for both profiles and all stages.

Figures 17 and 18 present the resulting thrust values of BADA 3, BADA 4 and INM (ANP) in function of H_p for the ICAO A Stage 5 profile.

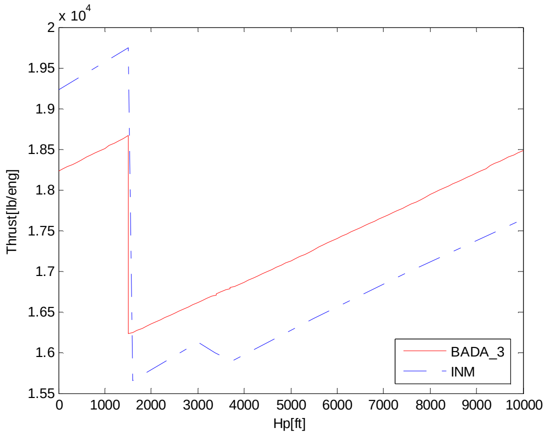

Figure 17. BADA 3 Thrust, ICAO A Stage 5

For ICAO A the results can be divided into three parts according to H_p and the error for both BADA families.

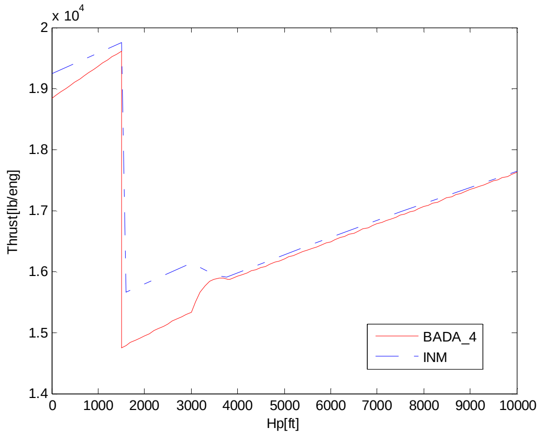

Figure 18. BADA 4 Thrust, ICAO A Stage 5

In the first part, up to 1500 [ft], the maximum take-off (MTKF) engine rating is assumed by ICAO A profile. Since MTKF is not supported by any of the BADA families, the maximum climb (MCMB) engine rating is used instead, and final values are multiplied by a constant value (1.33) to emulate a MTKF rating, as can be seen in Figures 17 and 18. Because of that approximation, the error in this part of the profile is not discussed here.

The second part ranges from 1500 to 4000 [ft] and is characterized by quite an important error for BADA 4. Note that this error is significantly decreasing from 3000 [ft]. This part will be explored in more details in the future, but since the aircraft reference performance data used in the identification were available from 1500 [ft] above, it may be explained as a boundary behaviour.

The third part, from 4000 to 10000 [ft], is characterized by an almost constant error for both BADA families.

The relative errors for the ICAO A profile are given in Table 3. The relative errors (%) for the first two parts (up to 4000 [ft]) and for the third part (above 4000 [ft]) or provided for both BADA families. The behaviour of the ICAO B profile is similar to the ICAO A profile for all stages and is not shown here.

Table 3. Relative Errors, ICAO A, BADA 3 and 4

Stage	**1**	**2**	**3**	**4**	**5**
BADA3	8/5.5	8/5.5	7.5/5.5	7.5/5.5	6/5.5
BADA4	8/0.5	8/0.5	7/0.5	7/0.5	5.5/0.5

This example shows that the independent use of the BADA Thrust model for both BADA families is possible. Up to 4000 [ft], important errors are expected because an unsupported engine rating is used in that segment, and because of lacking aircraft performance reference data for these altitudes. Above 4000 [ft], the improved accuracy of BADA 4 is demonstrated and the matching obtained in respect to INM is good. If the accuracy presented here for BADA 3 is acceptable for an application, then the BADA 3 Thrust model can also be used, albeit with a lower accuracy than BADA 4.

Conclusion

This paper provides an overview of the BADA APM with the focus on its ability to support modeling and simulation of complex flight instructions and operating regimes, such as economy climb, cruise and descent based on cost index, maximum range cruise, long range cruise, optimum altitude or maximum endurance cruise.

The optimization procedures and equations in which they derive are presented. Some results and examples are shown demonstrating the close relation between the qualities of the underlying models – realism and accuracy being the most important ones – and the quality of the results. It is shown that the use of BADA 3 with complex instructions is limited because of its insufficient level of realism and accuracy, whereas BADA 4 can be useful with complex instructions owing to its improved realism and accuracy.

The improvements included in BADA 4 enable an increase of accuracy in existing modeling and simulation applications, as well as the use of BADA in new application areas, such as strategic planning and optimizations.

The independent use of BADA 4 sub-models, such as the Thrust model, is demonstrated. It is shown that both BADA families may be used in this way, albeit with different levels of accuracy: BADA 4 may be used if high accuracy is requested, and BADA 3 only if low accuracy is acceptable. The independent use of BADA sub-models, their accuracy and usability will be further investigated.

EUROCONTROL shall continue its activities in the domain of APM. BADA 3 has been continuously maintained and improved in terms of number of aircraft supported and quality of aircraft instances. In parallel, research, development and implementation of BADA 4 have been undertaken. The first release of BADA 4 is planned for 2011. The appearance of BADA 4 will greatly help in facing the challenges of ATM. It is expected that the use of both BADA families together, BADA 4 with improved realism and accuracy, and BADA 3 with high coverage, will answer many challenges of today's and tomorrow's ATM.

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Email addresses

damir.poles@eurocontrol.int

angela.nuic@eurocontrol.int

vincent.mouillet@eurocontrol.int