Effect of aircraft self-merging in sequence on an airborne collision avoidance system

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This air traffic management research study analysed the interaction between a potential future airborne spacing application and an existing Airborne Collision Avoidance System (ACAS). The time-based airborne spacing application ‘merge behind’ was simulated in fast-time for a range of merge angles (45°, 90°, 135° and 180°), target spacing times (60 and 90 s) and altitudes (6,000 and 11,000 feet) under turbulent wind and extreme entry conditions. Trajectory pairs were analysed for potential collision alerting conditions using an ACAS simulator based on Traffic Alert and Collision Avoidance System (TCAS) II version 7 logic. Results show how, with realistic turn anticipation, the TCAS estimated time to go to Closest Point of Approach (CPA) decreased as merge angle was increased and as target spacing was reduced, but still remained above the Traffic Advisory (TA) and Resolution Advisory (RA) thresholds for the duration of all trials.

Nomenclature

| ACAS | = Airborne Collision Avoidance System |
| ADS-B | = Automatic Dependant Surveillance – Broadcast |
| AGL | = Above Ground Level |
| ASAS | = Airborne Separation Assistance System |
| ATM | = Air Traffic Control |
| CAS | = Calibrated Airspeed |
| CPA | = Closest Point of Approach |
| CDTI | = Cockpit Display of Traffic Information |
| GS | = Groundspeed |
| InCAS | = Interactive simulator for TCAS |
| RA/TA | = Resolution Advisory/Traffic Advisory |
| tau | = TCAS estimated time to go to CPA |
| TCAS | = Traffic Alert and Collision Avoidance System |
| TMA | = Terminal Control Area |

I. Introduction

A new allocation of tasks between controller and flight crew is envisaged as one possible option to improve air traffic management and in particular the sequencing of arrival flows. It relies on a set of new spacing instructions where the flight crew can be tasked by the controller to maintain a given spacing (in time or in distance) with respect to a designated aircraft. This task allocation, denoted airborne spacing, is expected to increase controller availability. This could lead to improved safety, which in turn could enable better quality of service and, depending on airspace constraints, more capacity. In addition, it is expected that flight crews would gain in awareness and anticipation by taking an active part in the management of their situation with respect to a designated aircraft. The motivation is neither to ‘transfer problems’ nor to ‘give more freedom’ to flight crew, but really to

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identify a more effective task distribution beneficial to all parties without modifying responsibility for separation provision. Airborne spacing assumes airborne surveillance (ADS-B) along with cockpit automation (Airborne Separation Assistance System, ASAS). No significant change to ground systems is initially required.

Particular airborne spacing applications are currently under investigation to help the air traffic controller sequence aircraft in zones of convergence such as the TMA (Terminal Control Area) and extended TMA surrounding an airport. Two new main kinds of spacing instruction ‘merge behind’ and ‘remain behind’ are being evaluated in air traffic controller and pilot in the loop real-time simulation based experiments. The ‘merge behind’ along with its variant ‘heading then merge behind’ instructions have been devised to assist the air traffic controller create sequences. Similarly the complementary ‘remain behind’ instruction is intended to help air traffic controllers maintain sequences.

Past studies have investigated both distance and time based airborne spacing of sequences of aircraft using fast time and real-time experiments (and references therein). Time based spacing has been found to have the following potential advantages over distance based spacing:

(i) allows an air traffic controller to issue only one spacing during changes of altitude and crossing sectors,
(ii) fits naturally with runway occupancy constraints,
(iii) allows an aircraft to follow the speed profile of the aircraft in front with small speed deviations and without cumulative upstream slowdown effects,
(iv) gives the pilot of the trailing aircraft a time delay to anticipate changes in the lead aircraft speed,
(v) produces smaller spacing errors when perturbed for given performance characteristics in the datalink such as latency, update rate and accuracy.

In addition, although wake vortex separation categories are currently expressed in terms of distance, there may be advantages in using decay times.

Before ASAS airborne spacing can be implemented in the ECAC (European Civil Aviation Conference) area, several questions remain to be answered. In particular, the issue of ACAS and ASAS interaction has to be addressed. ACAS II compliant equipment TCAS II version 7 is now part of the current European ATM System. Studies such as investigated performance of TCAS II Version 7 in UK airspace using simulations of London TMA airspace in particular to determine the rates of TAs and RAs. The IAPA (Implications on ACAS Performances due to ASAS implementation) project conducted a preliminary study of ACAS-ASAS interaction with a rough model of two merging aircraft. Results indicated little interaction expected for nominal conditions but aspects such as turbulent winds, pilot behaviour, closed-loop time based guidance and sensitivity analysis were not addressed.

The objective of this study is to quantify how close nominal and non-nominal encounters of aircraft performing a time-based airborne spacing ‘merge behind’ application come to triggering TCAS alerts, i.e. a sensitivity analysis to investigate margins of safety. Based on previous algorithms, this paper derives a time-based guidance law for the ‘merge behind’ application capable of merging aircraft under turbulent wind and extreme entry conditions. The guidance law was evaluated for non-nominal conditions using fast-time simulations. Resulting trajectories were analysed for potential collision alerting conditions using an ACAS simulator.

The paper is organised as follows: the airborne spacing ‘merge behind’ application is described, followed by the time-based spacing guidance law and ACAS description. Section “Apparatus” introduces the fast-time simulation environment: the MATLAB/Simulink platform, the aircraft model, wind model, atmosphere model and TCAS simulator. The evaluation method describes the operational test scenarios, metrics and range of experimental parameters used. Results on guidance law validation and interaction of airborne spacing with ACAS are presented as a series of time graphs followed by discussions. Main results are summarised and suggestions for future work are given in conclusion.

II. Airborne spacing ‘merge behind’

The time-based airborne spacing ‘merge behind’ application involves an air traffic controller instructing a pilot to select a neighbouring aircraft as a target on a Cockpit Display of Traffic Information (CDTI). An example of the phraseology developed is:

-Controller: “XYZ select target 1234”
-Pilot of XYZ: “XYZ, target 1234 identified, 8 o’clock, 30 miles”

Once the target 1234 has been selected, the air traffic controller can then instruct the pilot to merge behind the target at a given merge waypoint WPT ahead with a given spacing of 90 s. An example of the phraseology developed is:

-Controller: “XYZ, heading 270 then behind target merge WPT 90 seconds behind”
-Pilot of XYZ: “XYZ, merging WPT”

where the spacing is defined as a time in seconds.
An example of the ‘merge behind’ application is illustrated in Figure 1. The two aircraft, the lead (target) and the trailing aircraft, are flying straight to the same fixed merge waypoint. The solid arrows represent the current position and track angle of the aircraft, and the dashed arrows represent the desired positions of the two aircraft when the lead ‘reaches’ (it may not be overflown) the merge waypoint. By this point the spacing in time between aircraft must be within a defined tolerance from the desired spacing, and the aircraft should have similar speeds. After the waypoint the problem is similar to the in-trail following aircraft situation, i.e. each aircraft follows its own trajectory within a sequence maintaining the spacing between itself and the aircraft immediately in front.

![Figure 1: Airborne spacing ‘merge behind’ application.](image)

Conceptually, for the purposes of closed-loop guidance law design, this operational goal can be extended upstream of the merge waypoint by defining and minimizing a continuous time spacing error \( t_{\text{error}} \), at time \( t \). The spacing error \( t_{\text{error}} \) is defined as the difference between the elapsed time \( (t - t^*) \) since the lead aircraft was at the same distance from the merge waypoint as the trailing aircraft is currently and the desired time spacing \( t_{\text{spacing}} \):

\[
\begin{align*}
    t_{\text{error}}(t) &= t - t^* - t_{\text{spacing}} \\
    \text{where } t^* \text{ satisfies:}
\end{align*}
\]

\[
    d_{\text{lead}}(t^*) = d_{\text{trail}}(t)
\]

This time based spacing error was rewritten as an equivalent constant distance based expression. The equivalent distance based spacing error \( d_{\text{error}} \) was defined as the difference between the delayed position where the lead had been \( t_{\text{spacing}} \) seconds ago and the current position of the trailing aircraft.

\[
    d_{\text{error}}(t) = d_{\text{lead}}(t - t_{\text{spacing}}) - d_{\text{trail}}(t)
\]

### III. Time-based spacing guidance

The guidance law aims to establish a given spacing by the waypoint to a lead aircraft through speed adjustments as a pilot or as cockpit automation would do. The guidance law receives surveillance data from the lead aircraft and feeds the desired target calibrated airspeed input \( \text{CAS}_{\text{trail,desired}} \) into the aircraft model. The desired target altitude \( h_{\text{des}} \) is fed independently, and depends on the top of descent scenario. The following criteria were considered when designing the spacing guidance law:

(i) Spacing error at the merging waypoint should be small.

(ii) The trailing aircraft should have a similar speed to that of the lead aircraft for a smooth transition in to a ‘remain behind’ application after the waypoint.

(iii) The frequency of speed changes asked of the pilot in the trailing aircraft should be low enough to be operationally acceptable.
(iv) The speed profile of the trailing aircraft should be smooth with minimal speed deviations because it may itself be a target lead for another trailing aircraft behind.

The following spacing guidance law was derived using the above criteria:

\[
\text{CAS}_{\text{trail desired}} = \text{GS} \rightarrow \text{CAS}\text{ conversion}\left(\text{GS}_{\text{trail desired}}\right),
\]

where the desired CAS of the trailing aircraft \( \text{CAS}_{\text{trail desired}} \) is derived by converting the desired groundspeed of the trailing aircraft \( \text{GS}_{\text{trail desired}} \) to the equivalent CAS. The desired groundspeed of the trailing aircraft is based on the groundspeed of the lead aircraft where it was \( t_{\text{spacing}} \) seconds before, \( \text{GS}_{\text{lead}}(t - t_{\text{spacing}}) \), plus a corrective speed term derived from the spacing distance error \( d_{\text{error}} \) divided by the time to go before the lead aircraft arrives close to the merging waypoint \( t_{\text{lead to go}} \):

\[
\text{GS}_{\text{trail desired}}(t) = \text{GS}_{\text{lead}}(t - t_{\text{spacing}}) + \frac{d_{\text{error}}}{t_{\text{lead to go}}}
\]  

(1.4)

where \( t_{\text{lead to go}} \) is given by:

If \( d_{\text{lead}} \geq 2 t_{\text{spacing}} \text{GS}_{\text{lead}} \) then

\[
t_{\text{lead to go}} = \frac{d_{\text{lead}}}{\text{GS}_{\text{lead}}} - t_{\text{spacing}}
\]  

(1.5)

else

\[
t_{\text{lead to go}} = t_{\text{spacing}}
\]  

(1.6)

**Turn anticipation model**

Before arriving at the merge waypoint, both aircraft start to turn in order to achieve the new common trajectory. In reality, turn anticipation may vary depending on aircraft navigation system or pilot as well as aircraft speed and wind velocity. This study assumes that both aircraft roll to turn with a bank angle no greater than 25°. The new track is captured promptly, in a smooth manner and without any significant lateral overshoot. During, and after the turn, when both aircraft are flying along the same trajectory they have to maintain spacing. The guidance law used to maintain the in-trail spacing is similar\(^{11}\) to the merging guidance, and it is not detailed in this paper.

**IV. Airborne collision avoidance**

Since 1\(^{st}\) January 2000, in the European Civil Aviation Conference (ECAC) area all civil fixed-wing turbine-engined aircraft having a maximum takeoff mass exceeding 15,000 kg or a maximum approved passenger seating configuration of more than 30 are mandated to be equipped with ACAS II. From 1\(^{st}\) January 2005, the mandatory carriage of ACAS II will apply to all civil fixed-wing turbine-engined aircraft with a maximum takeoff mass exceeding 5,700 kg or authorised to carry more than 19 passengers\(^{2}\).

Based on Secondary Surveillance Radar (SSR) and Mode S technology, ACAS II equipment operates independently of ground-based aids and Air Traffic Control (ATC). Aircraft equipped with ACAS II have the ability to monitor other aircraft in the vicinity and assess the risk of collision by interrogating airborne transponders. Non-transponding aircraft are not detected\(^{2,14}\).

ACAS II compliant equipment TCAS II version 7 is an instrument integrated into other systems in an aircraft cockpit. It consists of hardware and software that together provide a set of electronic eyes so the pilot can ‘see’ the traffic situation in the vicinity of the aircraft. Part of the TCAS capability is a display showing the pilot the relative positions and velocities of aircraft up to 40 miles away. The instrument sounds an alarm when it determines that another aircraft will pass too closely to the subject aircraft. TCAS provides a backup to the air traffic control system’s regular separation processes\(^{6,14}\).
ACAS II can issue two types of alert: TA and RA. The collision avoidance logic is based on two concepts: sensitivity level and the time to go to CPA, ‘\(\tau\)’. The collision avoidance logic employs several distinct performance or Sensitivity Levels (SL), which vary as a function of altitude and which define the level of protection. The time to go to CPA, ‘\(\tau\)’ is calculated from range and range rate of the intruder plus a correction term \(D_{\text{mod}}\) (Distance Modification) for low closure rates. The critical ‘\(\tau\)’ thresholds for TAs and RAs depend on SL and vary as a function of altitude. An overview of the ‘\(\tau\)’ thresholds, \(D_{\text{mod}}\) and SL values is given in Table 1. A complete description of the TCAS logic can be found in\(^{14,24}\).

### V. Apparatus

The airborne spacing ‘merge behind’ application was simulated in fast-time using aircraft, pilot and wind models implemented with MATLAB/Simulink. Resulting trajectory pairs were analysed using a dedicated TCAS simulator (InCAS). Perfect airborne surveillance transmission quality of lead aircraft position and velocity to the trailing aircraft was assumed for both TCAS and ASAS i.e. continuous update rate, no delay, and perfect accuracy.

#### A. Aircraft model

The aircraft model includes the basic equations of motion, aerodynamic model, engine model, auto-pilot, auto-throttle control system, aircraft sensors and air-data model. The aircraft model is based on point mass equations of motion but with additional realistic rotational dynamics about the centre of gravity. The model includes lateral motion of the centre of gravity and dynamic characteristics of the engines. A detailed description can be found in the Appendix. An admissible speed envelope model based on physical limits like stall speeds and maximum airframe speeds is incorporated in the aircraft model. These limits may not be as conservative as airline normal operational limits.

#### B. Spacing director model

The desired CAS of the trailing aircraft \(C_{\text{trail,desired}}\) is passed through a quantiser to reduce the variations before presentation to the pilot model. This ‘spacing director’ logic takes the form:

\[
\text{if } |C_{\text{trail,in}}(t) - C_{\text{trail,out}}(t)| \geq 4 \text{knots} \text{ then } \quad C_{\text{trail,out}}(t) = C_{\text{trail,in}}(t) \quad (1.7)
\]

\[
\text{else } \quad C_{\text{trail,out}}(t) = C_{\text{trail,out}}(t - 1) \quad (1.8)
\]

The threshold of 4 knots for the quantisation was found to be a good compromise between accuracy and number of steps acceptable operationally. The final output is rounded to the nearest knot before presentation to the pilot model.

<table>
<thead>
<tr>
<th>Altitude</th>
<th>SL</th>
<th>Traffic advisory Range tau threshold (s)</th>
<th>Resolution advisory Range tau threshold (s)</th>
<th>Dmod (NM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 1000 ft</td>
<td>2</td>
<td>20</td>
<td>0.30</td>
<td>no RA</td>
</tr>
<tr>
<td>1000 – 2350 ft</td>
<td>3</td>
<td>25</td>
<td>0.33</td>
<td>15</td>
</tr>
<tr>
<td>2350 ft – FL050</td>
<td>4</td>
<td>30</td>
<td>0.48</td>
<td>20</td>
</tr>
<tr>
<td>FL050 – FL100</td>
<td>5</td>
<td>40</td>
<td>0.75</td>
<td>25</td>
</tr>
<tr>
<td>FL100 – FL200</td>
<td>6</td>
<td>45</td>
<td>1.00</td>
<td>30</td>
</tr>
<tr>
<td>&gt;FL200</td>
<td>7</td>
<td>48</td>
<td>1.30</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 1: Altitude dependent TCAS parameters.
C. Human pilot model

Pilot reaction to the above demand from the spacing director ($CAS_{trail\text{demand}}$) is modelled by a 5 s constant time delay based on observations of pilots in real-time simulations.

D. Wind model

The wind model is based on that of the Joint Aviation Requirements All Weather Operations (JAR-AWO) autoland certification process\textsuperscript{15}. In this model the mean wind speed is altitude dependent, and directly associated with the wind as measured at 30 feet AGL (Above Ground Level). The mean wind speed determines the turbulence intensity, and the wind velocity increases with altitude (Figure 2).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{turbulent_wind_vs_altitude.png}
\caption{Turbulent wind versus altitude - JAR-AWO model (20 knots wind at 30 feet AGL).}
\end{figure}

\textbf{Mean wind}

The magnitude of the mean wind increasing with altitude is defined by the following expression:

$$V_{\text{mean}}(h) = V_{30} \left( \frac{h}{10} \right)^{1/7},$$

where $V_{\text{mean}}$ is the mean wind speed (knots) measured at h metres AGL and $V_{30}$ is the mean wind speed (knots) at 30 feet AGL.

\textbf{Turbulence spectrum}

The turbulence model has a Gaussian distribution, conforming to the Dryden spectrum. The turbulence provided disturbances of the aircraft airspeed and angle of attack.

E. TCAS simulator

InCAS is an interactive fast-time simulator for analysing both real and synthetic TCAS II Version 7 encounters. InCAS simulates TCAS surveillance, the TCAS logic, TCAS-TCAS coordination, the cockpit annunciations and display of TCAS alerts, and has additional tools to analyse and represent TCAS data graphically\textsuperscript{13}.
VI. Method

A. Guidance validation

1. Operational scenario
To validate the guidance law the following scenario was simulated:
The lead and trailing aircraft were in descent from 29,000 feet to 5,000 feet, flying straight to the same fixed
waypoint. The lead aircraft track was due north and the trailing aircraft track angle due west. At the initial time the
lead aircraft was at 80 NM from the waypoint and constant speed 250 knots CAS. The trailing aircraft was
initialised at 93 NM from the waypoint with a speed of 250 knots CAS. Exceptionally in this guidance validation
scenario no turn anticipation was included in the lateral navigation system model in order to investigate worst case
closure rate. The scenario was conducted under realistic atmospheric conditions with turbulent winds from the north,
of mean strength 20 knots at 30 feet altitude and 52 knots at 30,000 feet altitude.

2. Metrics
The following metrics were used to evaluate the performance of the guidance law:
(i) Time based spacing error in seconds at the waypoint.
(ii) Groundspeed difference in knots between aircraft at the waypoint.
The intermediate speed profiles were also checked for operational acceptability as detailed in the four guidance
design criteria (see section ‘Time-based spacing guidance’).

B. Interaction between airborne spacing and ACAS
The tests involved a sequence of two aircraft on merging trajectories. Trajectories generated by MATLAB
models were recorded and used without further modification in the InCAS simulator. Both aircraft were: equipped
with TCAS II Version 7, fitted with Air Data Computers (providing fine altitude to own TCAS logic), and reported
altitude in 25 ft quanta.

1. Operational scenario
Two aircraft were flying along different merging trajectories at the same constant altitudes (two sets of trials for
6,000 and 11,000 feet). The lead aircraft track was due east and the trailing aircraft relative merge angle was varied
in an anti-clockwise direction between the values [45°, 90°, 135°, 180°]. At the initial time the lead aircraft was at
80 NM from the waypoint with a speed of 250 knots CAS. The trailing aircraft was initialised with the same speed
as the lead. For each merge angle and desired spacing, the trailing aircraft was initialised at the maximum admissible
distance (i.e. a maximum initial time error) such that it was still able to achieve the desired spacing at the waypoint
(see Table 2). The scenario was conducted with strong turbulent winds from the east of mean strength 20 knots at 10
m altitude. A tailwind for the lead was found to produce the smallest ‘tau’. Taking UK Meteorological Office data
for comparison, average annual wind speed in the London area is about 8 knots at 10 m altitude. On average the
London area experiences fewer than 5 days of gales a year where gale is defined as wind speeds of more than 34
knots for more than 10 minutes in a 24 hour period.

<table>
<thead>
<tr>
<th>Target spacing</th>
<th>Entry conditions</th>
<th>Merge angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial distance (NM)</td>
<td>45  90  135  180</td>
</tr>
<tr>
<td>60 s</td>
<td>Initial time error (s)</td>
<td>160 130 110  85</td>
</tr>
<tr>
<td></td>
<td>Initial distance (NM)</td>
<td>101 96  93  90</td>
</tr>
<tr>
<td></td>
<td>Initial time error (s)</td>
<td>220 160 140 115</td>
</tr>
</tbody>
</table>

Table 2: Entry conditions of trail aircraft function of spacing and merge angle.

Notice that the scenarios assumed an extreme operational situation, when two flows of aircraft, coming from
opposite directions (merge angle of 135° and 180°), are at the same flight level.
2. Metrics

A given set of entry conditions was deemed valid if they resulted in both the following two criteria being met:

(i) Spacing criterion: Time spacing error at the waypoint had to be less than 5% (i.e. 3 s for 60 s of spacing and 4 s for 90 s of spacing). See spacing guidance design criterion (i). This value is consistent with pilots in real time simulations being able to manually maintain spacing within 0.5 NM.

(ii) Speed criterion: Groundspeed difference between the aircraft at the waypoint had to be less than 10 knots. See spacing guidance design criterion (ii).

Due to the wind speed and direction, the maximum admissible entry position for the trailing aircraft varied between 101 NM from the waypoint (for 90 s of spacing, and 45° merge angle) and 88 NM from the waypoint (for 60 s of spacing and 180° merge angle), see Table 2.

TCAS ‘tau’ approximation

In MATLAB, the TCAS ‘tau’, was approximated by the following expression:

\[
\tau = r - \left( \frac{D_{\text{mod}}^2}{r} \right)
\]  

(1.10)

where \(r\) and \(\dot{r}\) are, respectively, the range and range rate of the intruder. The ‘Dmod’ is given in Table 1. This expression allowed a subset of MATLAB generated encounters to be selected for further detailed analysis using the real TCAS logic of the InCAS simulator. For each altitude and merge angle, the encounters with entry and wind conditions which produced the lowest ‘\(\tau\)’ were selected.

3. Experimental parameters

A Boeing 747 model (initial mass 271 tonnes) was used for the trailing aircraft type and a Fokker 100 model (initial mass 38 tonnes) for the lead aircraft type. This combination of aircraft types was considered to be the most testing case for two reasons:

(i) Wake vortex regulations allow closest minimum legal separation when a heavy aircraft follows a smaller aircraft (see Table 3).

(ii) Previous studies indicated that a heavy aircraft following a smaller one produced the largest spacing errors.\(^{12}\)

<table>
<thead>
<tr>
<th>Lead aircraft</th>
<th>Trail aircraft</th>
<th>Separation (NM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy Mass &gt;136 tonnes</td>
<td>Heavy</td>
<td>4</td>
</tr>
<tr>
<td>Medium</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Light</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Medium 7 tonnes&lt; Mass ≤ 136 tonnes</td>
<td>Heavy</td>
<td>3</td>
</tr>
<tr>
<td>Medium</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Light</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Light Mass ≤ 7 tonnes</td>
<td>Heavy</td>
<td>3</td>
</tr>
<tr>
<td>Medium</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Light</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3**: ICAO Wake Vortex Category Separations.

The following parameters were varied (Table 4):

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target spacing at merge waypoint</td>
<td>60 s, 90 s</td>
</tr>
<tr>
<td>Merge angle</td>
<td>45°, 90°, 135°, 180°</td>
</tr>
<tr>
<td>Aircraft altitude</td>
<td>6,000 and 11,000 feet</td>
</tr>
</tbody>
</table>

**Table 4**: Variable parameters.

6,000 and 11,000 feet were chosen to avoid ‘\(\tau\)’ threshold transition boundaries at FL50 and FL100.
4. **TCAS Analysis**

Sixteen merge encounter trajectories were filtered from the MATLAB based airborne spacing simulations using the minimum ‘tau’ criteria (see equation 1.10). These trajectories were analysed using InCAS to observe how the ‘tau’ behaved with time, the relation with thresholds and to note the time at which any TAs and/or RAs occurred.

### VII. Results

#### A. Guidance validation

The results of the MATLAB fast-time simulations are shown in Figures 3, 4 and 5. The maximum time-scale in the graphs corresponds to the time for the lead to reach the waypoint (~12 minutes). The guidance law produced small time spacing errors (Figure 3, less than 2 s at the waypoint) and similar groundspeeds (Figure 4, difference less than 15 knots).

![Figure 3: Spacing error between aircraft.](image1)

![Figure 4: Groundspeed of the lead and trailing aircraft.](image2)

![Figure 5: Comparison between the desired CAS given by the guidance law and the demanded CAS from the spacing director.](image3)

The performance of the spacing director model is shown in Figure 5. The desired CAS calculated by the guidance law (dotted line) was passed through the spacing director. The frequency of the CAS speed changes asked of the pilot (solid line) became low enough to be operationally acceptable, without any loss of stability.

#### B. Interaction between airborne spacing and ACAS

Figure 6 shows how ‘tau’ varies with merge angle for target spacings of 60 s and 90 s respectively in level flight at 6,000 feet. Figure 7 shows how ‘tau’ varies with merge angle for target spacings of 60 s and 90 s respectively in level flight at 11,000 feet. (Note that it is a feature of the TCAS logic that ‘tau’ is capped at its minimum value.) Each figure shows both TA and RA thresholds. The maximum time-scale in the graphs (horizontal axis) corresponds to the time when the two aircraft turned and both of them reached the new trajectory after the merge waypoint. The zero value on the horizontal axis corresponds to the time when ‘tau’ achieved its minimum value. Since the range
between aircraft is always significantly larger than Dmod, the difference between the variations of RA ‘\( \tau \)’ and TA ‘\( \tau \)’ is a fraction of a second (see equation 1.10). For clarity the figures show only the RA ‘\( \tau \)’ variation.

Figure 6: Variation of ‘\( \tau \)’ when target spacing 60 s and altitude 6,000 feet (left), when target spacing 90 s and altitude 6,000 feet (right).

Figure 7: Variation of ‘\( \tau \)’ when target spacing 60 s and altitude 11,000 feet (left), when target spacing 90 s and altitude 11,000 feet (right).

VIII. Discussion

As expected the margin between ‘\( \tau \)’ and TA thresholds reduces as merge angle increases and target spacing reduces. ‘\( \tau \)’, with turn anticipation, remained above the Traffic Advisory (TA) thresholds in all the cases investigated, however the margin between ‘\( \tau \)’ and the TA threshold in the worst case (target spacing 60 s at 11,000 feet), was only 1 s for 180° merge angle (see the summary results in Table 5).

<table>
<thead>
<tr>
<th>Altitude (feet)</th>
<th>Spacing (s)</th>
<th>‘( \tau )’ : minimum values (s)</th>
<th>TA thresholds (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>45°</td>
<td>90°</td>
</tr>
<tr>
<td>6,000</td>
<td>60</td>
<td>142</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>139</td>
<td>91</td>
</tr>
<tr>
<td>11,000</td>
<td>60</td>
<td>141</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>206</td>
<td>91</td>
</tr>
</tbody>
</table>

Table 5: Minimum values for ‘\( \tau \)’. 
Note that for all trials the minimum legal separation was never infringed. The minimum spacing between aircraft (~5 NM for 60 s of target spacing and ~8 NM for 90 s of target spacing) was obtained when both aircraft passed the waypoint.

Figure 8 is a plan view of the aircraft tracks showing how both aircraft anticipated turns for a merge angle of 180°. The coordinates of the merge waypoint are [0, 80 NM]. This encounter represents the worst case investigated, merge angle 180°, 60 s target spacing, and aircraft co-altitude at 11,000 feet. The line connecting the trajectories indicates the aircraft positions at time of minimum ‘tau’. As shown, in this extreme case, ‘tau’ does come close to triggering a TA (see Table 6). Notice that the minimum ‘tau’ occurred at the lead mid turn point when the lead aircraft was closest to the merge waypoint (Figure 8).

![Figure 8: Lead and trail trajectories for a merge angle of 180° and 60 s of spacing.](image)

<table>
<thead>
<tr>
<th>Altitude</th>
<th>FL110</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum range</td>
<td>4.9 NM</td>
</tr>
<tr>
<td>Traffic range ‘tau’ threshold</td>
<td>45 s</td>
</tr>
<tr>
<td>Range tau threshold</td>
<td>30 s</td>
</tr>
<tr>
<td>Minimum range ‘tau’</td>
<td>46 s at 14:07</td>
</tr>
<tr>
<td>Range at 14:07</td>
<td>7.1 NM</td>
</tr>
</tbody>
</table>

Table 6: Lead TCAS results for 180° merge angle and 60 s of spacing.

*Effect of no turn anticipation before merge waypoint*

The previous results assumed smooth turn anticipation of the merge waypoint. But how early could alerts occur if turns were not anticipated before the merge waypoint? Table 7 shows, for encounters without turn anticipation, how ‘tau’ dropped below the TA thresholds for 60 s of target spacing with 135° and 180° merge angles. It was noted that an RA on the 180° merge angle would probably have occurred if the scenario were extended just 1 s further.

<table>
<thead>
<tr>
<th>Altitude (feet)</th>
<th>Spacing (s)</th>
<th>Time before merge waypoint if TA occurred (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45°</td>
<td>90°</td>
</tr>
<tr>
<td>6,000</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>-</td>
</tr>
<tr>
<td>11,000</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7: TAs without turn anticipation.
IX. Conclusions

The interaction between a potential future airborne spacing application and an existing Airborne Collision Avoidance System (ACAS) was investigated. Using a time-based guidance law the airborne spacing ‘merge behind’ application was simulated in fast-time for a range of merge angles, target spacing times and altitudes under turbulent wind and extreme entry conditions. Pairs of trajectories were analysed for potential alerting conditions using an ACAS simulator based on TCAS (Traffic Alert and Collision Avoidance System) II version 7 logic.

Results confirm that the TCAS estimated time to go to Closest Point of Approach (CPA), ‘Such’, decreased as merge angle was increased and as target spacing was reduced.

For 90 s target spacing, ‘Such’ remained at least 18 s above the Traffic Advisory (TA) thresholds, when both aircraft anticipated turns. Even without turn anticipation no TAs occurred before the lead aircraft overflew the merge waypoint. This is consistent with the hypothesis that the airborne spacing ‘merge behind’ application for target spacing over 90 s should not normally induce TCAS alerts when smooth turns are anticipated and aircraft follow their planned tracks.

For 60 s target spacing and merge angles less than 90°, ‘Such’ remained at least 17 s above the TA thresholds, when both aircraft anticipated turns. Even without turn anticipation, no TAs occurred before the lead aircraft overflew the merge waypoint. However, for merge angles above 135°, ‘Such’ margins varied from 1 s down to 1 s (at 180°). Without turn anticipation several TAs occurred for merge angles above 135°. It should be noted though, that the co-altitude assumption for high merge angles like 180° is very extreme and operationally aircraft would normally be vertically separated.

This analysis based on extreme entry conditions and observing TCAS ‘Such’ behaviour may be useful when designing airspace and procedures for ASAS ‘merge behind’ real-time experiments.

To complement this study it would be interesting to perform a similar robustness analysis for different combinations of turn anticipation times and lateral deviations from the planned lead and trail tracks. Compounding effects of airborne surveillance transmission quality such as update rate, latency and accuracy could also be investigated. Other future research could include a more detailed analysis for sequences of aircraft in descent.

Appendix: Aircraft model

For this study an aircraft model was required with realistic behaviour along typical descent profiles, including speed and heading changes, and intermediate altitude steps. The aircraft model is divided into two parts:

- The aircraft dynamics models the actual physics of the system.
- The pilot model is a combined representation of the aircraft auto pilot system and to a certain extent of the pilot actions on it.

For the aircraft dynamics, the following general assumptions are made:

- Flat, non-rotating earth.
- Standard atmosphere.
- Fully co-ordinated flight. The sideslip angle $\beta$ is always zero and there is no side force.

The equations of motion used for the aircraft model are based on the three-dimensional point-mass differential equations, as found in many references\(\textsuperscript{20, 21}\). The total set of differential equations results in 7 state variables, $[\gamma V h \phi \psi x_{east} x_{north}]$, where: $\gamma$ is the flight path angle, $V$ the true airspeed, $h$ vertical distance or altitude, $\phi$ is the bank angle, $\psi$ the heading angle, $x_{east}$ the east position and $x_{north}$ the north position and $m$ the aircraft mass. Because the aircraft mass is not considered to be constant, the equations of motion are complemented by an eighth equation, describing the loss of mass due to the fuel flow ($Q$) of the aircraft. The final set of equations are given hereafter:

$$
\dot{\gamma} = \frac{L + T \cdot \sin \alpha}{m \cdot V} \cdot \cos \phi - \frac{g}{V} \cdot \cos \gamma
$$

(2.1)
\[
\begin{align*}
\dot{V} &= \frac{T \cdot \cos \alpha - D}{m} - g \cdot \sin \gamma \\
\dot{h} &= V \cdot \sin \gamma \\
\dot{\phi} &= p \\
\dot{\psi} &= \frac{2 \cdot \tan \phi}{V} \\
\dot{x}_{\text{east}} &= V \cdot \cos \gamma \cdot \cos \psi - V_{\text{wind}} \cdot \cos \chi_{\text{wind}} \\
\dot{x}_{\text{north}} &= V \cdot \cos \gamma \cdot \sin \psi - V_{\text{wind}} \cdot \sin \chi_{\text{wind}} \\
\dot{m} &= -Q
\end{align*}
\]

Here, \(D\) is the drag, \(T\) the engine thrust, \(\alpha\) angle of attack, \(\chi_{\text{wind}}\) and \(V_{\text{wind}}\) are the wind direction and speed, \(L\) is the lift, \(p\) is the roll rate and \(g\) is gravity. A normal flight regime, is considered in this study, therefore \(\alpha\) is relatively small, and in (2.2) \(\cos \alpha\) can be approximated to 1. Further, in (2.1), the term \(T \cdot \sin \alpha\) can be considered as negligible in comparison with the lift contribution. This simplifies (2.1) and (2.2) to:

\[
\begin{align*}
\dot{y} &= \frac{L}{mV} \cdot \cos \phi - \frac{g}{V} \cdot \cos \gamma \\
\dot{V} &= \frac{T - D}{m} - g \cdot \sin \gamma
\end{align*}
\]

Differential equations (2.3) to (2.10) constitute then the basic equations of motion of the aircraft model. Aerodynamic forces are modelled using an estimate of the aircraft trimmed aircraft polar, with an extension to take into account the effects of Mach-drag rise. The Mach-drag rise component is usually a function of Mach number and lift coefficient. A 2-dimensional look-up table is used to model the aircraft polar.

Thrust is computed from a given thrust over weight ratio for a given aircraft, by multiplying this ratio by a percentage thrust command and the maximum take-off mass of the aircraft type at hand. The thrust over weight ratio is calculated from a two-dimensional look-up table, as function of Mach and pressure altitude. Thrust characteristics used in the model are typical for high by-pass turbofan aircraft. Due to the fact that thrust is calculated as a dimensionless thrust over weight ratio, the thrust model can be adapted easily to various aircraft types, without significant changes to the thrust model. By using a calibration factor (ranging from plus or minus 20%) the model can therefore easily be adapted to any aircraft type.

Acknowledgements

This CoSpace project study was sponsored jointly by: the EUROCONTROL Experimental Centre (EEC); the European Air Traffic Management Programme (EATM) of EUROCONTROL; and the European Commission Directorate General for Energy and Transport Trans-European Networks programme. The authors wish to thank the IAPA project team (QinetiQ, CENA, Sofreavia, EUROCONTROL HQ & EEC) and Garfield Dean for their helpful review comments.

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