Conflict resolution in presence of uncertainty: A case study of decision making with dynamic programming

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Abstract

From a system perspective, one important characteristic of the air traffic system, is the uncertainty or even the lack of knowledge of future states (aircraft trajectories). In a such context, deciding when an action (to solve a conflict) has to be performed is an open issue. However, since the knowledge of future states (aircraft trajectories) is dynamically refined and updated, the air traffic system can be considered as a dynamic system. In this context, attempting to identify an "optimal" time to manoeuvre (and the manoeuvre itself) can be formulated as a problem of optimal control of a dynamic system. It has been shown that this class of problem can be modelled by the Hamilton-Jacobi-Bellman equation. The dynamic programming approach could be used to solve this equation for a dynamic system. As a first attempt to take advantage of this perspective, the paper proposes an initial problem statement (including the HJB equation and a cost function), along with a possible dynamic programming algorithm to solve it. Some simulations have been conducted to stress the impact of different cost functions and level of knowledge about surrounding aircraft.

Introduction

From a system perspective, one important characteristic of the air traffic system, is the uncertainty or even the lack of knowledge of future states (aircraft trajectories). In a such context, deciding when an action (to solve a conflict) has to be performed is an open issue. Conflict detection and resolution in presence of uncertainty has been widely studied [KY97], in particular considering uncertainty related to aircraft positions and trajectories, e.g. [EPIE97][BAB+00]. The uncertainty considered are attached to an aircraft isolated from surrounding traffic, and modelled the influence of wind [CGJ+00], aircraft modelling, errors in control and navigation [YK97].

Surrounding aircraft having to manoeuvre to solve conflicts is another cause of uncertainty. The uncertainty resulting from the possible manoeuvres of a conflicting aircraft has been investigated in [YK97]. This form of uncertainty naturally increases as the traffic density increases. For instance, the effect of traffic density on the stability of conflict resolution has been evaluated in [BSLG00] through a measurement of the "domino effect". This allowed to compare different conflict resolution techniques, e.g. centralised vs. decentralised [KPB00].

Investigating the appropriate time to manoeuvre considering multiple aircraft potentially in conflict thus appears as a question still to be addressed. Different approaches could be used to address this question, such as game theory or semidefinite programming. An other approach is selected here. Indeed, since the knowledge of future states (aircraft trajectories) is dynamically refined and updated, the air traffic system can be considered as a dynamic system. In this context,
attempting to identify an "optimal" time to manoeuvre (and the manoeuvre itself) can be formulated as a problem of optimal control of a dynamic system [Ber95]. It has been shown that this class of problem can be modelled by the Hamilton-Jacobi-Bellman (HJB) equation [BD97] [Bar94]. The dynamic programming approach could be used to solve this equation for a dynamic system [Ber95].

As a first attempt to take advantage of this perspective, we present here an initial problem statement (including the HJB equation and a cost function), along with a possible dynamic programming algorithm to solve it. The paper is organised as follows: the first section introduce the problem statement; the second one presents the simulation environment; the third one discusses the possible ways to define a cost function, and the last section shows preliminary results.

Problem statement
Identifying the "optimal" time to manoeuvre and the manoeuvre itself is formulated as minimising a given cost, i.e. a mathematical function which defines what is an undesirable outcome. In addition to the cost function, operational constraints are also considered, and formulated as inequalities or domain definitions.

The decision making is a step by step process, based on dynamic programming. This means that at each stage, decisions or controls, are based on both the current cost and on the future expected cost. This method ensures an “optimal” control of a dynamic system over a finite number of stages (i.e. a finite horizon time) [Ber95]. However, it should be noticed that this decision making process is not proposed as a new conflict resolution algorithm, but rather a mean to investigate different decision making strategies.

Notations
The following notations are used in the paper.

$T$: horizon time.

$u_k$: control variable or $k^{th}$ manoeuvre.

$\bar{u}_k$: scalar value of the control variable.

$y$: state variables vector.

$(x_1, x_2)$: position state variables.

$\alpha$: heading state variable.

$l$: integral part of the cost function.

$c$: cost due to the manoeuvre.

$\Omega$: available airspace for own.

$U$: set of admissible manoeuvres.

$\partial_{t_k}$: Dirac.

$f$: part of the dynamic function.

Problem formulation
The problem lies in minimising a cost function depending of the state variables $y=(x_1, x_2, \alpha)^T$ and of the control variable $u$. There are three state variables: two of them are the position coordinates in a 2D plan, and the third one is the current heading. There is only one control variable because of the restriction on the resolution manoeuvre which is a combination of heading changes. It can be noted that the control variable is an impulsive one.

$$\min_u \left[ \int l(t, y(t)) dt + \sum_k c(u_k) \right]$$

where $y(t) \in \Omega$, $t \in [0; T]$, $u_k \in U$, $k = 0, ..., n$.

The dynamic equations are:

$$\dot{y}(t) = f(y(t)) + \sum_k \partial_{t_k} (t) \bar{u}_k$$

$$y(0) = (x_1(0), x_2(0), \alpha(0))^T$$

To respect the operational constraint of no conflict, some areas of the airspace are forbidden to the own aircraft during a certain time. Moreover, own aircraft cannot exit the airspace except by the exit zone (both geographic and temporal). Therefore the available airspace for own aircraft (denoted $\Omega$) is not the complete airspace. The dynamic equations are:

$$x_1(t) = x_1(0) + tv \cos \alpha$$

$$x_2(t) = x_2(0) + tv \sin \alpha$$

$$\alpha(t) = \alpha(0) + \sum_{t_k \leq t} \bar{u}_k$$

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Or equivalently, in differential form:

\[
\begin{aligned}
\dot{\xi}_1(t) &= -v \sin \alpha \\
\dot{\xi}_2(t) &= v \cos \alpha \\
\dot{\alpha}(t) &= \sum_k \partial_{\alpha_k}(t) u_k
\end{aligned}
\]

In addition, to ensure desired exit position and time, two other constraints (denoted as final constraints in dynamic programming) are considered:

\[
\begin{aligned}
\|X(T) - X_{\text{ref}}(T_{\text{ref}})\|^2 &\leq K_1^2 \\
T_{\text{ref}} - T &\leq K_2
\end{aligned}
\]

with \(T\) the time at which the own aircraft exit the area, \(T_{\text{ref}}\) the desired exit time, \(X(T)\) the final position of own aircraft, and \(X_{\text{ref}}(T_{\text{ref}})\) the desired exit position. Let denote \((P)\) the problem including these two last constraints, the problem \((P_0)\) and the dynamic equations.

**Dynamic programming principle and Hamilton - Jacobi - Bellman equation**

Let \(V\) be the solution of the problem \((P)\). Then, according to the dynamic programming principle, the value function \(V\) satisfies for all \(\tau>0\):

\[
V(y) = \inf_u \left[ \int_0^T l(y(t),t)dt + \sum_{k,t_f<\tau} c(u_k) + V(y(\tau^-)) \right]
\]

Let \(M\) be an operator as:

\[
M : V(x) \longrightarrow \min_u \{ MV(x) \}
\]

\[
MV(x) = \min_{u \in U} [c(u) + V(x+u)]
\]

The value of the problem \((P)\) is solution in the viscosity meaning, of the HJB equation [Bar94], [BCD97]:

\[
\max_{u \in U} \left[ -V(x,t) - H(x,t,D_x V(x,t)), \right] V(x,t) - MV(x,t) = 0
\]

with: \(H(x,t,p)=l(x,t)+f(x,t).p\)

Let be: \(w(x,t)=V(x,T-t)\).

Therefore:

\[
\max_{u \in U} \left[ w(x,t) - H(x,t,D_x w(x,t)), \right] w(x,t) - Mw(x,t) = 0
\]

After a time discretisation, it comes:

\[
\max \frac{\bar{V}^{k+1}(x) - \bar{V}^k(x)}{\Delta t} - H(x^k, t^k, \bar{V}^k(x)) , \bar{V}^{k+1}(x) - M\bar{V}^k(x) = 0
\]

Equivalently, the time discretised HJB equation can be written as follows:

\[
\bar{V}^{k+1}(x) = \min \left[ \bar{V}^k(x) + \Delta H(x^k, t^k, D\bar{V}^k(x)), M\bar{V}^k(x) \right]
\]

After this discretisation in time, the HJB equation is also discretised in space. Thereafter, at each time step, the conflict resolution algorithm will solve the resulting equation. That is to say, the algorithm will try to find a control \(u_k\) that minimises the cost. This cost can be decomposed in two distinct parts: the transition cost and the final cost. The transition cost represents the cost associated to the manoeuvre, while the final cost represents the estimated cost to reach the end position. It should be noticed that if the new state variables (position or heading of the aircraft) are not on one point of the grid, the cost is estimated from the neighbouring points, with a numeric interpolation function. This function is a continuous, piecewise affine function.
Simulation environment

The simulation will consist in an aircraft having to avoid conflicts with multiple intruder aircraft. Three different scenarios will be considered:

1. Own aircraft knows in advance all intruders’ future trajectories.
2. Own aircraft only knows the intruders’ future trajectories but only valid for the next time step.
3. Own aircraft knows the initial intruder trajectories but it is "aware" of their possible re-planning.

There will not be other uncertainty that those resulting from the possible re-planning.

The airspace considered is an horizontal square of 125\times 125\text{Nmi}. The simulation lasts 15min.

The intruders are flying at constant speed chosen randomly between 350kts and 480kts.

A traffic generator creates intruders flight plans as follows: an initial position at time 0 (which can be inside or outside the airspace), an initial heading, and a set of new headings together with their flight time associated. A new aircraft is then introduced, referred as own aircraft. This aircraft has the property to be in conflict with all the intruders inside the airspace, and to be the only one allowed to manoeuvre. Its flight plan is created similarly to intruders' ones, except on two points: its starts on the boundary of the airspace, and it is not allowed to go outside. Own aircraft has the constraint to avoid conflicts with heading manoeuvres, to follow as much as possible its flight plan and to be inside a defined zone at the end of the simulation.

Uncertainties are introduced in intruders’ future trajectories as follows. At each time step the intruders can change, with or without previous advice, their announced flight-plans. This will be referred as a re-planning. To respect a certain logic the re-planning will not change the past and preserve the final destination but not the arrival time at this destination. It will no longer assure conflict-free with own aircraft during a given initial period, if no previous notice has been delivered. The re-planning consists of the position change of ways points (WPs). WPs are moved proportionally to their time situation. That is to say, the WPs before and at the beginning of the re-planning and the WP at the end of the flight plan stay unchanged, the others move more especially as they are closed to the position at the middle of remaining flight duration. An illustration is given in Figure 1.

Cost function study

This section tries to classify the different costs included in the resolution manoeuvre. It raises questions regarding the definition of the
optimality of a solution, and the impact of a cost function conflict resolution. It also attempts to define an evaluation framework in order to compare different cost functions.

Motivation
The core of the conflict resolution problem lies in finding a solution (manoeuvre, initial and final times) said optimal regarding given criteria and constraints. As the cost function defines what is "optimal", the choice of cost function directly determines the solution characteristics. In fact, different conflict resolution “modes” can be envisaged, corresponding to different operational situations. For example: a normal mode with conflict avoidance and respect of the flight plan used for a classic conflict resolution; an emergency mode with only conflict avoidance used for critical situations and a saturated airspace; an accident mode with re-planning of all intruders' flight plans to ensure own a direct route used in case of accident. It is important to outline that the only difference within the conflict resolution algorithms between the use of these different modes is the choice of the cost function. Therefore the understanding of the impact on solution, of each cost function component, is a major issue to ensure conflict resolution flexibility and efficiency.

Classification of costs
A classification of the different possible costs induced by a resolution manoeuvre is proposed and discussed. A distinction is made between operational, safety and stability costs. (It should be noticed that a same cost can be classified into different categories.)

- **Operational**: additional economical costs due to a conflict resolution manoeuvre. They have no impact on the safety of the manoeuvre. Examples of possible indicators: cost of deviation (fuel consumption and additional delay), and cost in term of comfort for passengers (e.g. how many successive turns?).

- **Safety**: parameters that may impact directly or indirectly on the safety of a resolution manoeuvre. Examples of possible indicators: is it difficult or not to execute the resolution manoeuvre; how many space is available to execute it, that is to say at which distance from the others aircraft will be the new trajectory?

- **Stability**: parameters that may impact on the stability of the resolution manoeuvre. A solution will be defined as stable if it ensures two points. First, the decision of manoeuvre is not taken too early and is really necessary. Second, own aircraft will not create any new conflict by manoeuvring. Examples of possible indicators: how many aircraft in conflict situation are closed to the new resolution trajectory? At which distance? What is the impact on the rest of the traffic?

The different costs may be classified in another way. Because some questions regarding the way to associate the individual cost, and the way to share a common goal may be raised.

Costs impacts analysis
In order to illustrate the impact of the each cost, the decision making process described previously will be used to solve a same conflict situation, with the same constraints but with different costs. It is assumes that own aircraft knows initially all the future intruder positions. The cost-function used here is very basic. It can penalise the deviation from own initial flight plan, the number of heading changes and the amplitude of the heading changes.

With cost-function 1, a perfect balance between a limited number of trajectory changes and a limited deviation from the initial flight plan was found. The conflict resolution’s manoeuvre starts early and can be viewed as “heading then remain”. With cost-function 2, the deviation from the initial flight plan is limited. That is the reason why the aircraft waits the last moment to start its resolution manoeuvre. To allow for a minimal deviation in order to execute its resolution manoeuvre, a margin of 5Nmi is given. That is to say, a deviation of 5Nmi at maximum is less penalised than a deviation upper than 5Nmi. There is no penalisation on the
heading change. With cost-function 3, only the number of manoeuvres and their amplitudes are considered. There is no penalisation on the deviation from the initial flight-plan, but there is still the constraint on the exit position. That is why the aircraft limits the number of heading changes to avoid the conflict and then to reach the exit position. With cost-function 4, a non-well tuned cost-function example is presented.

Figure 2 illustrates the different conflict resolution manoeuvres.

A numerous others dimensioning parameters can be added to this example of cost-function. The key point will be to predict and understand their impacts and interactions.

**Complexity and stability of a solution**

One possible improvement will be a cost-function component that “describes” the complexity of a situation. This would be particularly useful with high density or aircraft to ensure the stability of a solution. The ideal complexity parameter will help the algorithm to prevent the effect of the unplanned trajectory changes in the surrounding traffic ("domino effect"). An indicator of the traffic density could be a first step in this direction.

**Decision making process**

To estimate the importance of the complexity parameter, a test was done with the same conflict resolution algorithm, the same cost-function, the same constraints, but different levels of information on the intruders’ future trajectories.

There are three levels of information:

- **L1**: own aircraft has a short-term view (one time-step ahead) of intruders’ future trajectories.
- **L2**: own aircraft has a perfect knowledge of the future intruders’ intents, but intruders can re-plan their trajectories every time they want, without advise.
- **L3**: own aircraft has a full knowledge of intruders’ future trajectories.

The scenario is built as described previously in section "Simulation environment". In order to limit the computation time, the number of intruders has been limited to two.

The cost-function restricts own aircraft headings (number and amplitude). Moreover it limits the deviation from the initial flight-plan. Note that because of the constraints (e.g. conflict-free, exit point, heading change limited to 45°) no solution may exist.

First, different situations corresponding to the desired scenario are randomly chosen to evaluate the impact of the three levels of information. The outcomes considered are simple: conflict is solved or not, and if a conflict is solved the exit position is monitored. Table 1 gives the result of this experimentation.

As expected, the performances of the algorithm improve with the level of information. It is interesting to notice that with L1, the percentage of correct exit positions is particularly high compared to the others results. This can be explained as follows. Own aircraft has a short-term view of intruders’ future trajectories. Therefore its best strategy is to follow as much as possible its initial flight-plan to the detriment of heading changes. Moreover, for L2 and L3 the aircraft can only manoeuvre in heading, so it cannot compensate its delay with a velocity.
change even if the correct exit position is targeted.

<table>
<thead>
<tr>
<th></th>
<th>Conflict</th>
<th>No conflict</th>
<th>Correct exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>43.33%</td>
<td>56.67%</td>
<td>41.18%</td>
</tr>
<tr>
<td>L2</td>
<td>40%</td>
<td>60%</td>
<td>11.11%</td>
</tr>
<tr>
<td>L3</td>
<td>22.5%</td>
<td>77.5%</td>
<td>22.58%</td>
</tr>
</tbody>
</table>

Table 1: Comparison of the 3 levels of information

The results obtained with L2 could be improved significantly if the intruders give an advice before changing their flight-plan.

Second, the same initial scenarios are used to compare L1 and L2. That is to say, to compare the perfect knowledge of intruders’ flight-plan before any re-planning and the resulting resolution, to the partial knowledge of intruders’ intents. Table 2 gives the results.

<table>
<thead>
<tr>
<th></th>
<th>Conflict</th>
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</thead>
<tbody>
<tr>
<td>L2</td>
<td>40%</td>
<td>60%</td>
<td>11.11%</td>
</tr>
<tr>
<td>L3</td>
<td>20%</td>
<td>80%</td>
<td>58.33%</td>
</tr>
</tbody>
</table>

Table 2: Comparison of the conflict resolution with and without uncertainties

The results are similar in table 1 and table 2, except for the percentage of correct exit for L3.

It is interesting to notice that in 10% of the cases, the re-planning has complicated the situation, in the sense that no resolution had been possible. This can be due to the new complexity of a situation, or it can be due to the lack of time to avoid the new conflict.

The gap between the percentage of successful resolutions with a perfect knowledge of intruders’ future trajectories and with the partial knowledge of these trajectories modulo uncertainties due to re-planning is of 20%. The main question will be to reduce this gap with the help of new indicators in the cost-function as a density or a complexity indicator. The idea would be to compare the conflict resolution manoeuvre obtained with different time horizon knowledge of intruders’ trajectories. As we have done here, and then to tuned new indicators to bring the manoeuvres nearer to the one obtained with a total knowledge of intruders’ trajectories.

**Conclusion**

This paper presented an initial attempt to use the principle of optimal control for dynamic systems. An initial problem statement along with a possible dynamic programming algorithm have been proposed. The tool allows to evaluate different decision making strategies. It has been highlighted that in the simulations conducted, the use of cost purely based on individual aircraft are not sufficient. The results and the experiments of this study will be re-used in a future work. This will consist in testing, with the decision making process described in this paper, different density and complexity indicators. In a first time, this will be done by adding corresponding terms in the cost function and by comparing the same scenarios with different horizon time of knowledge on intruders’ trajectories. Then in a second time, it will be tested with adding other navigation uncertainties than the re-planning uncertainties.

**References**


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