Abstract

This paper addresses the theoretical issue of the uncertainty modelling to enhance conflict estimation for air traffic control. One of the important aspect is to identify and evaluate the impact on the positioning error of its different components: position, velocity, and acceleration error. To set up such a model, the major difficulty identified is to make explicit the dynamic of aircraft position that includes uncertainties. A possible enhancement of the classical probabilistic model of error by introducing a dynamic dimension to the position error model is proposed. This new position error model is generic and can thus be used for different probabilistic methods. Moreover, as shown in the paper, this new model can be tuned to reproduce the results used in reference and validated on real traffic.

Introduction

Considering the air traffic growth, the major challenge facing air traffic control (ATC) is to enhance air traffic capacity while providing safety improvements. A possible option to address this challenge is to provide advanced conflict detection capabilities, to be used on the ground to assist the controller or on the air to support delegation of separation assurance to the pilot. This option raises many questions at different levels. It raises technical questions, typically for airborne applications, on the way to transmit the appropriate surveillance information among aircraft. It also raises questions regarding human factor aspects, particularly how this facility would impact on working methods, how to present the information to the controller or to the pilot?

This paper addresses the theoretical issue of the uncertainty modelling to enhance conflict estimation. One of the important aspect is to identify and evaluate the impact on the positioning error of its different components: position, velocity, and acceleration error. To set up such a model, the major difficulty identified is to make explicit the dynamic of aircraft position that includes uncertainties.

The field of conflict detection has been widely studied so far, both from ground and airborne perspectives, using different methods [11], and evaluation methodologies [3, 12]. The probabilistic approach proposed in [1, 2] seems to constitute a good candidate, since it allows a straightforward modelling of uncertainty, along with a fine modelling of position error for both conflict detection and separation monitoring. Furthermore, with this method, different assumptions and flight scenarios can be implemented easily. In addition, this method was validated by simulations and experimentation on real traffic.

This paper will present a first set of enhancements, mainly regarding the modelling of position uncertainty. Different models will be proposed and discussed. The tuning of these models by reproducing the results obtained in [1, 2] will constitute an important step of this study. It will enable the comparison between the models and the estimation of their accuracy.

The paper is organised as follows. First a state of the art is proposed. Then, a widely used probabilistic approach is outlined. Finally, the
new aircraft position error modelling is presented.

State of the art

Inherently, trajectory prediction is uncertain because of different types of error such as errors in wind and aircraft modelling, errors in control, and navigation, or re-planning due to human intervention (e.g. ATC instruction). Therefore, the trajectory prediction becomes more uncertain with time. The different errors can be taken into account or not to predict aircraft trajectories.

Each type of error can be modelled by various ways. However a same model can be used for different types of error. Depending on the way chosen to model the trajectory prediction error, there are different methods to detect a conflict [11].

A classification that relies on the level of modelling of errors is presented. The different levels of error modelling are defined by the way aircraft trajectory is predicted. Three types of trajectory predictions are described here: geometric, worst-case, and probabilistic. The corresponding conflict definition is given and some corresponding classical conflict detection methods are presented.

Geometric

A first step in conflict detection is to select for a geometric method. This method uses nominal trajectory without uncertainty. More precisely, it uses the initial trajectory prediction without taking into account re-planning and trajectory uncertainty.

- From aircraft current states and nominal trajectory without uncertainty, the Closest Point of Approach (CPA) can be computed easily. Then a comparison between the minimum predicted separation (also denoted as distance at CPA) and the separation minima determines if a conflict will occur [6].

- This method can be made a slightly more sophisticated. Instead of looking at only a pair of aircraft positions, all pairs of positions can be compared to the separation minima. An easy way to do this is to use a method called Fast-study [8]. That can be decomposed as follows. The aircraft nominal trajectories without uncertainty are defined by straight-line segments between points in 4D space that allow to take into account the turns and the altitude changes. An ordered sequence of times at trajectory points for each pair of aircraft is arranged. Each pair of segments will be put through the conflict detection test. This test involves computing the distance-squared separation at the beginning, end and midpoint of the remaining interval. Since this norm is a quadratic function of time and given these three points, the interval for which the distance is less than the allowed separation – if any – can be found with a simple routine.

Worst case

A second step in conflict detection is to take into account uncertainties by considering increasing levels of modelling.

- In the worst-case trajectory predictions, the positions of aircraft are represented by areas of uncertainties. This can be applied to straight flight paths. The areas of uncertainties are growing linearly with time. Then the corresponding conflict detection method can be assimilated to the first geometric method shown previously. However an extension of the conflict definition is needed. In fact, the aircraft is no longer represented by a single point but by an area. In that case, a conflict will occur if the minimum separation between the two areas of uncertainties is lower or equal to the separation minima [4].

- For the case of multiple-conflict detection using a worst-case trajectory prediction, this last method can be adapted [9]. To simplify the computation, some conditions can be relaxed and then the problem is rewritten in the form of a Linear Matrix Inequalities (LMI) problem.

- If all possible aircraft positions are considered during an entire flight (turns and altitude changes included) some refinements can be added [10]. Then the possible positions of the two aircraft are represented by areas of uncertainties. Similarly to the method associated to the worst case trajectory prediction for straight flight: a conflict will occur if the minimum separation between the two areas of uncertainties is lower or equal to the separation minima.
The construction of the uncertainty areas that takes into account turns, is as follows. An aircraft position is represented by a point at the initial time. However by adding uncertainties parameters, the point becomes a segment in the uncertainty direction, for example the speed direction. The first and last points of this segment (the aircraft position with uncertainties) correspond respectively to the aircraft position when flying at the minimum and maximum possible speeds. When changing direction the segment becomes a parallelogram that increases with speed direction. When changing direction again, the parallelogram becomes a polygon, and so on.

Probabilistic

A third step in conflict detection is to take into account uncertainty by weighting all possible aircraft positions. Therefore, the prediction of future trajectories becomes probabilistic. The conflict definition changes: a conflict occurs when the conflict probability is upper than a given threshold.

- The possible aircraft position can be weighted with a probability density functions, as a Gaussian [1].
- A new type of detection is needed. The two position errors have to be considered for a pair of aircraft, represented by two co-variance matrices. They are combined into a single equivalent co-variance. Then, a co-ordinate transformation is proposed that transforms the combined error co-variance into a standard form of a unit circle. Finally, an analytical solution for the conflict probability can be found.
- This last probabilistic method [1] can be refined by taking into account various uncertainty parameters and associated probability density functions such as heading change or avoidance response latency [5].
- An other way to model the uncertainties is to use random processes [13], and then the method becomes stochastic. Nevertheless, the conflict definition remains identical.

Rationales

The use of nominal trajectories without uncertainty to predict future aircraft positions is a time-limitation for the validity of the conflict detection. Indeed the accuracy of the prediction gets lower with time. Hence, the geometric method becomes unsafe or inefficient very quickly.

The use of worst-case trajectory predictions, may include a high rate of false alerting or a high rate of missed alerting. False alerting and missed alerting rates depend on the choice of the uncertainty areas bounds. Indeed, on one hand, a too large area may include low probability aircraft positions thus leading to false alerting. On the other hand, a too restrictive area may exclude high probability aircraft positions thus leading to missed alerting. The choice of the uncertainty areas bounds is a trade-off between efficiency and safety. However, all aircraft positions in the uncertainty areas are considered as having the same probability. Hence the compromise can not be totally satisfying.

In the probabilistic case all uncertainties are taken into account and weighted. As with the use of the worst case prediction, a trade-off must be identified between safety and efficiency. This trade-off arises in the choice of the probability threshold to set off the conflict alarm. However, with an appropriate model of error and a judicious choice of threshold, an accurate and efficient detection can be performed.

The study presented in this paper, is based on a widely used probabilistic method. This method was validated by simulations and experimentation on real traffic [1, 2].

A widely used probabilistic method

This section will describe the method [1, 2] used by Erzberger and Paielli to estimate the probability of conflict in the simple case of straight flights in two dimensions. This method, is based on a combination of deterministic trajectory prediction and stochastic conflict analysis to achieve reliable conflict detection.

Two errors of positions have to be considered for a pair of aircraft, represented by two co-variance matrices. First, the two prediction error co-variances are combined into a single co-variance of the relative position.
this combined co-variance is assigned to one of the aircraft, referred to as the "stochastic" aircraft, and the other aircraft, referred to as the "referenced" aircraft, can be regarded as having no position uncertainty. Third, the "stochastic" aircraft is fixed, only the "referenced" one is moving with a velocity equal to the relative velocity. Fourth, a coordinate transformation is done that transforms the combined error co-variance into a standard form of a unit circle. Fifth, some assumptions are made to limit the potential conflict zone to an infinite rectangular band in the direction of the relative velocity. Sixth, the analytical solution for the conflict probability is computed by integrating the probability density functions on the band.

In this method, the errors made on position are normally distributed with a constant error rate that linearly grows with time. That is to say, the position error results from a velocity error. More precisely the cross track root mean square (rms.) error is constant and the along track rms. error starts at zero and grows linearly with time. This gives the following equations:

\[ x(t) = v_N t + C t N(0,1) \]  \hspace{1cm} (1) \\
\[ \text{var}(C t N(0,1)) = \text{var}(x(t)) = C^2 t^2 \]  \hspace{1cm} (2)

with \( x(t) \) the along track position at time \( t \), \( v_N \) the nominal velocity, \( C \) the constant along track rms. error, \( N(0,1) \) the Gaussian distribution and \( \text{var}(x(t)) \) the variance of the along track position. This model of position error will be considered, as a reference for this study. More precisely, it will help to tune the proposed models of position error.

It should be noticed that the modelling of velocity error used results from a statistical study and the experimental results founded on this modelling, were validated on different live data [3]. Therefore, these results constitute a good illustration of reality and will serve as a reference for the modelling in this work.

To reproduce the reference results a set of numerical examples of conflict probabilities and related quantities are generated as a function encounter geometry. The aircraft speeds are 480kts (≈ 247m/s) in every cases. The conflict separation distance is 5nmi. The cross-track rms. error is 1nmi (1852m), and the along-track rms. error started at zero to grow linearly at a rate of 15kts (≈ 7.7m/s). Wind-error cross correlation between aircraft is not modelled.

Figure 1 shows the effects of minimum predicted separation as a function of time to CPA, for 90deg path-crossing angle.

For small prediction times, the co-variances are small and the conflict probabilities are a strong function of minimum predicted separation. That is to say, near the CPA, the conflict probability depends almost on the predicted minimum separation. For larger prediction times, the co-variances grow and the conflict probabilities become a weaker function of minimum predicted separation. The conflict probabilities converge and asymptotically approach zero as prediction time increases.

Figure 1: Effects of minimum predicted separation with Erzberger and Paielli’s model.

**New error modelling**

A suitable error modelling is an important component of probabilistic conflict detection methods. Indeed it determines the quality of trajectory prediction and therefore directly impacts on the accuracy of the conflict detection. It had been seen that according to Erzberger and Paielli the position error could be modelled as normally distributed with a constant rate that linearly grows with time. This position model cannot be directly linked to the corresponding velocity model or acceleration model. Among that, a question appears: how can the dynamic model of the position error be made explicit?
Position error models
This part will focus on the position error model. New models will be proposed and discussed.

A first possible model is to take the position error as resulting from an error on the velocity. This error can be due, for example, to navigation instrument errors. In this part, this velocity error is chosen to be modelled as a Brownian noise.

The Brownian movement is usually used by physicists to model the trajectory of a small particle. In a liquid this particle makes uncoordinated movements resulting from the perpetual shocks that it puts up with. A process is Brownian if it is continuous, its growths are independent and stationary and if each component of the process is a centred Gaussian (that is to say with a mean equal to zero).

It can be noticed that with this last property of a Brownian process, the Gaussian position error model can be found again by stopping the Brownian motion at a fixed time. A Brownian on the velocity gives the following equations:

\[ dx(t) = v_N \, dt + \sigma_1 \, dW_1(t) \]  
\[ x(t) = v_N \, t + \sigma_1 W_1(t) \]  
with \( x(0) = 0 \)

\[ \text{var}(\sigma_1 W_1(t)) = \text{var}(x(t)) = t \sigma_1^2 \]  
(5)

with \( \sigma_1 \) the rms. error, \( dW_1 \) a white noise, and \( W_1 \) the corresponding Brownian.

A second model can be a position error resulting from an error on the acceleration. It can happen for example when there is wind or with navigation instruments errors.

It can be noticed that with the normally distributed position error model, the effects of winds are not taken into account. However the parameter used as the velocity error rate has been found with statistical studies. That is why an error on acceleration due to winds can exist in this modelling without being explicit.

By modelling the acceleration as a Brownian process the experimental results of Erzberger and Paielli can be found again. That gives the following equations:

\[ dv(t) = \sigma_2 \, dW_2(t) \]  
\[ v(t) = v_N + \sigma_2 \, W_2(t) \]  
(8)

\[ x(t) = v_N \, t + \sigma_2 \int_0^t W_2(s) \, ds = v_N \, t + Z(t) \]  
(9)

with \( x(0) = 0 \)

with \( \sigma_2 \) the rms. error, \( dW_2 \) a white noise, and \( W_2 \) the corresponding Brownian.

The integral of a Brownian is a Gaussian with the following variance [7]:

\[ \text{var}(Z(t)) = \text{var}(x(t)) = \frac{t^3}{3} \sigma_2^2 \]  
(10)

Mixing these two models and the “reference” model, with using different proportions (here \( \alpha \), \( \gamma \) and \( \beta \)) leads to a set of new possibilities. Indeed the position error can be written as follows:

\[ x(t) = v_N \, t + \alpha \sigma_1 W_1(t) + \beta C t \, N(0,1) \]

\[ + \gamma \sigma_2 \int_0^t W_2(s) \, ds \]

\[ \text{var}(x(t)) = \alpha^2 \, t \sigma_1^2 + \beta^2 \, t^2 \, C^2 + \gamma^2 \frac{t^3}{3} \sigma_2^2 \]  
(12)

This position error model gives an infinity of combinations with three degrees of freedom \( (\alpha, \gamma, \beta) \) and six adjustable parameters (\( \alpha \), \( \beta \), \( \gamma \), \( \sigma_1 \), \( \sigma_2 \), \( \sigma_3 \)).

Comparison of these models
The results of Erzberger and Paielli presented in the previous part will be called “reference results” in this part.

The different models of error can not be directly compared since some involve probability density functions and others involve stochastic processes. However their impacts on the conflict detection will be studied and compared. To test the new position error models, the experimental results seen in the figure 1 are reproduced with the same data, except those concerning the error on the along-track position. The test will be done by tuning of the model to duplicate the reference results.
results. Therefore some values of the position error rms. will be found by using the following method.

This method is based on the conflict probability function properties. For a minimum predicted separation substantially less than minimum allowed separation, the conflict probability starts at unity and decreases monotonically as a function of prediction time. The effect of larger error growth rates is to cause the conflict probability to decrease more rapidly. Symetrically for a minimum predicted separation substantially greater than the minimum allowed separation, the conflict probability starts at zero and increases to some maximum value, and finally decreases back towards zero. The effect of larger growth rates is to cause the conflict probability to increase more rapidly and then after the maximum, to decrease more rapidly [1, 2].

When the conflict probability at CPA and 30min to CPA were equal to those found with the reference results and the behaviour of the curves were identical, the error rates were chosen to be presented here. Of course a different choice could have been done, for exemple the models could have been tuned with the conflict probability at CPA and 10 or 20 min to CPA.

By tuning only one curve for one value of predicted minimum predicted separation, all the others curves are also tuned. This is logical and proper to the conflict detection used.

First, the position error is the result from an error on the velocity. It can be due, for example, to navigation instrument errors.

Figure 2: Effects of minimum predicted separation with the velocity error modelled as a Brownian.

Figure 2 presents the effects of minimum separation with a position error resulting from a velocity error. This velocity error is chosen to be modelled as a Brownian process. The rms. error on velocity is here equal to 325m/s or 632kts. So it is superior to the speed of the aircraft (247m/s or 480kts). This is not a proportion physically realistic. Moreover this model detects worse the conflict than the reference model. That is to say it detects the conflict later. So it is not the dynamic model corresponding to the reference model. However it can be used to partially characterise the error of position.

Second, the position error is the result from an error on the acceleration. It can happen for exemple when there is wind or with navigation instrument errors.

Figure 3: Effects of minimum predicted separation with the acceleration error modelled as a Brownian.

With the exception of the along-track error rate rms. (here 0.31m/s²) and using the same data in figure 1, the computation of the effect of minimum predicted separation with this modelling gives almost the same conflict detection (figure 3). Nevertheless the conflicts are better detected. That is to say, they are detected on average 5mn earlier than in the reference curves (figure1). This could be explained as follows. The aircraft navigation accuracy is taken into account more precisely, so the error of position made can be estimated earlier.

If an other path crossing angle is chose there is no effect on the tuning results.
Table 1: Comparison between the different models

<table>
<thead>
<tr>
<th>Min predicted separation</th>
<th>Time to CPA</th>
<th>0min</th>
<th>10min</th>
<th>20min</th>
<th>30min</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0nmi</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.95</td>
<td>0.69</td>
</tr>
<tr>
<td>2.5nmi</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.82</td>
<td>0.62</td>
</tr>
<tr>
<td>5.0nmi</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.48</td>
</tr>
<tr>
<td>7.5nmi</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.18</td>
<td>0.30</td>
</tr>
<tr>
<td>10nmi</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.30</td>
<td>0.15</td>
</tr>
</tbody>
</table>

To summarise and illustrate the differences between the models, the conflict probabilities computed with each position error model are compared in table 1 at different times and at different minimum predicted separations, with in first position the value with the reference model, in second position the value with the velocity error resulting from a Brownian noise, and in last position the acceleration error resulting from a Brownian noise.

Combination of models

It can be interesting now to combine the two more efficient models in order to study if and how they can be weighted to reproduce the result presented in figure 1.

Figure 4: Effects of minimum separation with a normal position error combined with a Brownian noise on the acceleration.

Figure 4 shows the effect of minimum predicted separation with a mixed model composed of a normally distributed position error with a constant rate (here 4m/s = 7.77kts) that linearly grows with time and an error resulting from a Brownian noise on the acceleration (here with a rms. = 0.26m/s²).

As expected the quality of the conflict detection is slightly better than in figure 1 and little worse than in figure 3. Naturally, this is proportional to the weights of each model.

This model has two different degrees of freedom and four adjustable parameters. They can be regulated depending on the aircraft dynamics and flight guidance characteristics.

Figure 5: Effects of minimum separation with a mixed position error model.

A mixed of the tree models presented in this paper can be available and provides equivalent results. Figure 5 shows an example with a mixed model composed of a normally distributed position error with a constant rate (here 1.5m/s = 2.9kts) that linearly grows with time, an error resulting from a Brownian noise on the velocity (here 5m/s = 9.5kts) and an error resulting from a Brownian noise on the acceleration (here 0.31m/s²).

This mixed model has three different degrees of freedom and six adjustable parameters. As
for the previous model they can be tuned depending on the aircraft dynamics and flight guidance characteristics.

Other possible tunings

Other tunings than the one used in this paper are possible.

If the position error model is tuned to reproduce the conflict probability values at CPA and 10min (respectively 20min) before CPA, the new curves are converging on the reference ones. During the 10min (respectively 20min) before CPA, the new curves are closer to the reference ones than those using the tuning at 30min to CPA. However the result properties remain the same. Before this period, the new curves are distant to the reference ones. The conflicts are worse detected by the model resulting from a Brownian on the acceleration, and better detected by the model resulting from a Brownian on the velocity. This behaviours can be explained mathematically by the time power in the models (2 for the reference model, 1 for the model resulting from a Brownian on velocity, 3 for the model resulting from a Brownian on the acceleration).

Robustness

To test the validity of the results, some curves with different error rms. values were computed and compared. Figures 6 and 7 show the effect of minimum predicted separation with the error rms. divided by two. Figures 8 and 9 show the effect of minimum predicted separation with the error rms. multiplied by two. In the figures 6 and 8, the position error model is the reference one. In the figures 7 and 9, the position error is resulting from an error on the acceleration modelled as a Brownian process.

As with the previous models, the conflicts are detected earlier with the error on the acceleration than with the reference model. This sensitivity analysis assures the robustness of the position error model resulting from a Brownian noise on the acceleration.

Figure 6: Effects of minimum separation with the error rms. divided by two and the reference model.

Figure 7: Effects of minimum separation with the error rms. divided by two and the error on acceleration as a Brownian.

Figure 8: Effects of minimum separation with the error rms. multiplied by two and the reference model.
Discussion

Different position error models have been tuned to give an accurate conflict estimation. Beyond, an appropriate modelling of uncertainty should be a compromise between three considerations: (1) a position error normally distributed with a constant rate that grows linearly with time, (2) a position error resulting from a velocity error modelled as a Brownian process, and (3) a position error resulting from an acceleration error modelled as a Brownian process. A combination of the three models presented in this paper is a possible solution. This model has three different degrees of freedom. To weight each model, and to fix the rms. values, the aircraft and their own dynamics and flight guidance characteristics should be considered. This would assure a more accurate modelling thus improving conflict detection.

Therefore, three key points that can benefit directly from this new error model have been identified. The first one is to identify, integrate and model all the uncertainties that affect “significantly” trajectory predictions. The second point is to provide capabilities not only for conflict detection but also for monitoring potential or solved conflicts, typically through the estimation of minimum distance. The third point identified is to allow the modelling of uncertainty under different assumptions and for different applications and scenarios. For instance, two typical cases are of interest: on one side aircraft flying with an auto-pilot and transmitting flight states, and on the other, aircraft flying with a Flight Management System (FMS) and transmitting full intent trajectories. That will be part of the future work.

Conclusion

This paper has proposed a possible enhancement of the classical probabilistic model of error by introducing a dynamic dimension to the position error model. This new position error model is generic and can thus be used for other probabilistic methods.

As shown in the paper, this new model can be tuned to reproduce the results used in reference and validated on real traffic.

References


