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FOR THE SAFETY OF AIR NAVIGATION

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**EUROCONTROL EXPERIMENTAL CENTRE**

# STOCHASTIC CONFLICT DETECTION FOR AIR TRAFFIC MANAGEMENT

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**TITLE:**

**STOCHASTIC CONFLICT DETECTION FOR AIR TRAFFIC MANAGEMENT**

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**Abstract:**

Considering the air traffic growth, the major challenge facing Air Traffic Control is to enhance air traffic capacity while providing safety improvements. A possible option to address this challenge is to provide some advanced conflict detection capabilities.

This paper deals with the appropriate modelling of uncertainty to provide reliable conflict estimation. It presents a new position error model for conflict detection. Usually the position error is modelled as normally distributed with a constant rate that linearly grows with time. Here it is modelled as resulting of a Brownian noise on the velocity and on the acceleration which should be more appropriate. The experimental results of H. Erzberger and R. Paielli are used to tune these new models. Then an extension of their conflict detection method for trajectory with heading and velocity changes is exposed. At the same time a reflection on wind effects is done.

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# Stochastic Conflict Detection for Air Traffic Control

## Summary

Considering the air traffic growth, the major challenge facing Air Traffic Control is to enhance air traffic capacity while providing safety improvements. A possible option to address this challenge is to provide some advanced conflict detection capabilities.

This paper deals with the appropriate modelling of uncertainty to provide reliable conflict estimation. It presents a new position error model for conflict detection. Usually the position error is modelled as normally distributed with a constant rate that linearly grows with time. Here it is modelled as resulting of a Brownian noise on the velocity and on the acceleration which should be more appropriate. The experimental results of H. Erzberger and R. Paielli are used to tune these new models. Then an extension of their conflict detection method for trajectory with heading and velocity changes is exposed. At the same time a reflection on wind effects is done.

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# ACRONYMS

- ADS-B : Automatic Dependant Surveillance Broadcast
- AIRSAW : Airborne Situational Awareness (ODIAC Task force)
- AOC : Airline Operation Centre
- A/P : Automatic Pilot or Auto-pilot
- APO : Aircraft performance and operation
- ASAS : Airborne Separation Assurance system
- ATC : Air Traffic Control
- ATCo : Air Traffic Controller
- ATM : Air  Traffic Management
- CFMU : The Central Flow Management Unit in Brussels
- CNS : Communication, Navigation and Surveillance
- CPA :  Closest Point of Approach
- CRCO : The Central Route Charges Office
- dCPA : distance at CPA, or minimum separation distance
- EACAC : Evolutionary Air-ground Co-operative ATM Concepts
- EATMS : The European Air Traffic Management System
- EEC : The EUROCONTROL Experimental Centre in France
- EFR : Extended Flight Rules
- ESCAPE : EUROCONTROL Simulation Capacity

- FFAS : Free Flight Airspace
- FMS : Flight Management System
- FREER : Free Route Experimental Encounter Resolution
- FS : Flight State
- GNSS : Global Navigation Satellite System
- IANS : The Institute for Air Navigation Services in Luxembourg
- MAS : Managed Airspace
- MUAC : The Maastricht Upper Area Control Centre
- nmi : Nautical Mile
- NUC : Navigation Uncertainly Category (position, velocity, wind)
- OCD : The Operational Concept Document
- ODIAC : Operational Development of Initial Air/Ground data Communications
- PHARE : Program for Harmonised ATM Research in EUROCONTROL
- rms. : Root main square
- RNAV : Area Navigation
- RVSM : Reduced Vertical Separation Minima
- $s_c$ : minimum allowed distance between the aircraft, or separation minima.
- tCPA : time before the CPA.
- TCAS : Traffic Alert and Collision Avoidance System

- TCP : Trajectory Change Point
- TIS-B : Traffic Information Service-Broadcast
- TV : Target Value

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# 1. INTRODUCTION

## 1.1. PRESENTATION OF THE SUBJECT

The role of Air Traffic Control (ATC) is: to prevent collision, to expedite and to maintain an orderly flow of air traffic.

One of the techniques used to avoid collision is to maintain separation of defined safety zones bounding each aircraft i.e. conflict avoidance. The separation method used in this paper is based on searching for the minimum predicted separation, which can be mathematically represented by the notion of Closest Point of Approach (CPA). With the CPA, it can be known if the minimum required separation is respected by the aircraft during its flight, and if a conflict will occur. This is a fundamental notion present in most conflict detection methods. Sometimes the use of CPA is not explicit, but may still be underlying. Furthermore, the CPA search is very important because it gives useful information for ensuring and monitoring the separation. It also has been used in collision avoidance, for example in the Traffic Collision Avoidance System (TCAS).

When the predicted distance at CPA is smaller than or equal to the separation minima, it will lead to a conflict. Different ways exist to compare the CPA and the minimum required separation. They all depend on the aircraft future trajectory predictions.

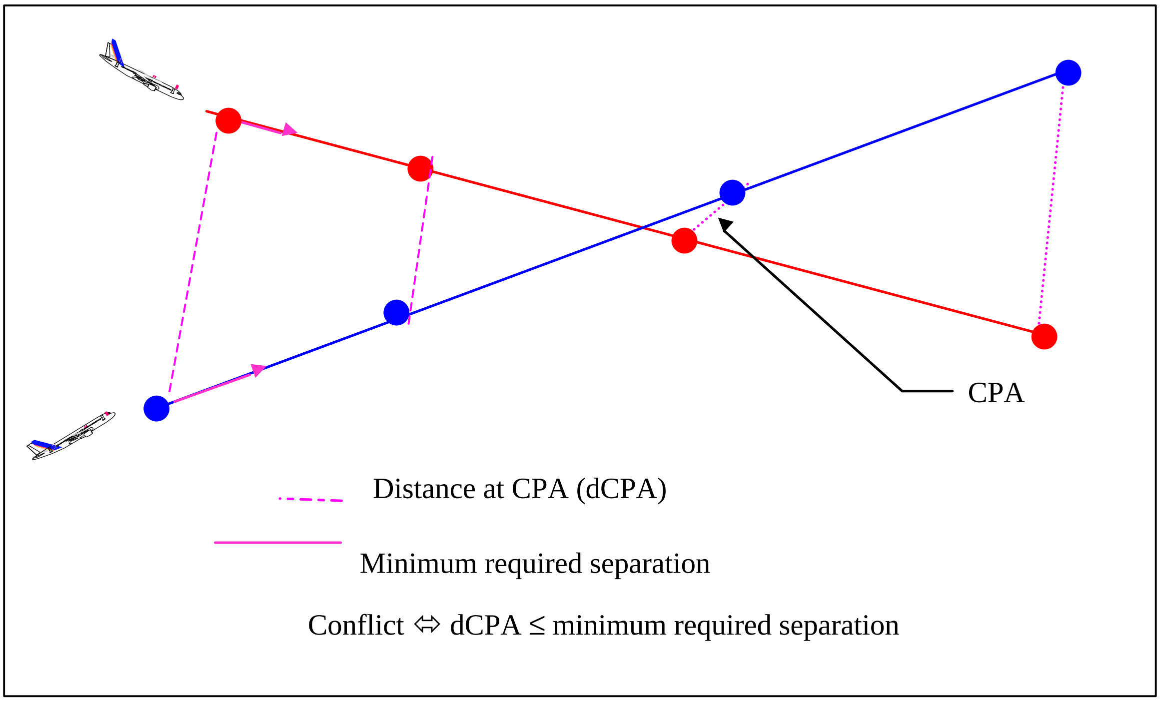


**Figure 1: representation of a conflict in 2D**

Figure 1 shows a representation of a conflict in 2D. It also illustrates the notion of CPA. The CPA can be defined as follows. It is the relative position of two aircraft when the range between them is a minimum. This definition is a geometrical one. In the following study some ways to extend the notion of CPA will be presented. Once the CPA is determined, it can be deduced :

- Position to CPA
- Distance to CPA
- Time before CPA
- Time when the separation is lost, if a conflict occurs

## 1.2. THE LOGIC OF CONFLICT DETECTION

The article "Survey of Conflict Detection and Resolution Modeling Methods" written by J.K. Kuchar and L.C. Yang [13] serves as a basis for the following two paragraphs.

To prevent collisions from either a human point of view (ATCo – Air Traffic Controller) or a system point of view, similar schemes can be applied. At a general level, these schemes can be divided into several phases, shown in figure 2.

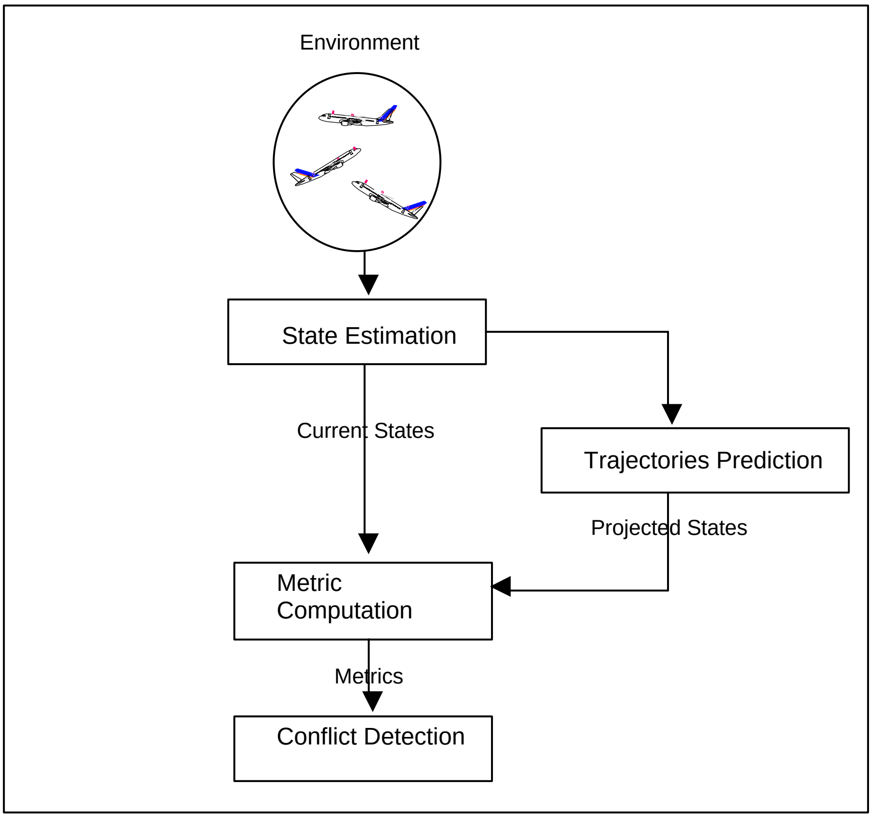


**Figure 2: Conflict Detection**

To detect a conflict for a pair of aircraft, the ATCo collects their state information. These states provide an estimate of the current traffic situation. Due to sensor errors, there is generally some uncertainty in the values of the states.

Then, a prediction of future aircraft trajectories is performed. This prediction may be based only on current states or may be based on additional information like a flight plan. As with the current state estimation there are usually some uncertainties in the estimation of the future trajectory. The prediction can be inexact, because of errors in : wind modelling, aircraft modelling and navigation errors. The farther in advance trajectories are predicted, the more uncertain those predictions are.

Finally, information regarding the current states and the predicted states can be combined to derive metrics in order to predict whether a conflict will occur. This metrics can be just the CPA and the separation minima, or they can also be information about the data's uncertainties. Thereafter, one can detect if a conflict will occur.

## 1.3. SYNTHESIS OF CONFLICT DETECTION METHODS

Inherently trajectory prediction is uncertain because of errors in wind and aircraft modelling, control and navigation errors, or re-planning due to human intervention (e.g. ATC instruction). The trajectory prediction becomes more uncertain with time. For that purpose, there are many ways to detect a conflict [11] and different ways to predict the future trajectories.

A classification can rely on the level of modelling of errors and how they derive from each other. Criteria for differentiating between current methods will be presented. Then their basic principles will be identified. Finally an evaluation and a comparison will be tried.

- A basic way to detect conflict *without* taking into account any *uncertainty* is to use a *geometric method*.
  - From aircraft current states and *nominal* trajectory predictions, the Closest Point of Approach (CPA) can be computed easily. Then a comparison between the minimum predicted separation (distance at CPA) and the separation minima determines if a conflict will occur [6].
  - This method can be made a slightly more sophisticated. Instead of looking at only the pair of aircraft positions, all pairs of positions can be compared to the separation minima. An easy way to do that is to use a method called *Fast-study* [8]. This method can be decomposed as follows. The aircraft *nominal* trajectories are defined by straight-line segments between points in 4D space that allow to take into account the turns and the change of altitude. First an ordered sequence of times at trajectory-points for both the trajectories is arranged. Every pair of segments will be put through the conflict detection test. This test involves computing the distance-squared separation at the beginning, end and midpoint of the remaining interval. This is a quadratic function of time and given these three points the interval for which the distance is less than the allowed separation – if any – can be found with a simple reusable routine.
- Another step is to take into account *uncertainties* by considering *different levels of accuracy.*
  - The *worst-case Trajectory Predictions* can be applied first to *straight flight paths*. That is to say, the positions of aircraft are represented by areas of uncertainties, which are growing linearly with time. Then the conflict detection method can be assimilated to the first geometric method shown previously. But an extension of the conflict definition is needed. In that case, a conflict will occur if the minimum separation

between the two volumes of uncertainties is lower or equal to the separation minima [4].

- For the case of *multiple-conflicts detection* using a *worst-case* prediction, this last method can be adapted [9]. To simplify the computation, some conditions can be relaxed and then the problem is rewritten in the form of a Linear Matrix Inequalities (LMI) problem.
- If *all* possible aircraft *positions* are considered during an *entire flight* (turns and change of altitude include), some refinements can be added [10]. Then the possible positions of the two aircraft are represented by *volumes of uncertainties*. As in the worst case trajectory prediction for straight flight, an extension of the conflict definition is used. A conflict will occur if the minimum separation between the two volumes of uncertainties is lower or equal to the separation minima.

  The construction of the uncertainty areas that takes into account turns, is as follows. An aircraft is represented by a point at the initial time. But the point becomes a segment in the uncertainty direction, for example the speed direction. The first and last points of this segment correspond respectively to the aircraft position when it flies at the minimum and maximum, possible speed. When changing direction the segment becomes a parallelogram that increases with speed direction. When changing direction again, the parallelogram becomes a polygon, and so on.
- If now *all possible aircraft positions are weighted*, then the prediction of future trajectories becomes *probabilistic* [1]. So a new kind of detection is needed. The two position errors have to be considered for a pair of aircraft, represented by two co-variance matrices. They can be combined into a single equivalent co-variance. Then, a co-ordinate transformation is proposed that transforms the combined error co-variance into a standard form of a unit circle. Finally, an analytical solution for the conflict probability can be found. Even if this method cannot be reduced in the checking of the CPA, this last one is used to simplify the computation.
- This last *probabilistic* method can be refined by taking into account various *other uncertainty parameters* and associated probability density functions such as heading change or avoidance response latency [5].

## 1.4. THE APPROACH CHOSEN

In this part, the limitations of the different methods presented previously are exposed. Then the further work done in this study will be presented and put in perspective with the other existing approaches.

The use of nominal trajectories (i.e. without uncertainty) to predict future aircraft positions can be a time-limitation for the validity of the conflict detection. As the computation of predicted aircraft positions does not take into account the possible uncertainties, the validity is only for a short time. To be operational this method needs frequent updates.

For the use of worst-case trajectory predictions, the false alerting rate can be high. Because for the computation of the conflict detection, the very small probability positions can be chosen.

In the probabilistic case all uncertainties are taken into account and weighted. The difficulties of this method come from the validity and the precision of error models. For this study a probabilistic approach will be chosen. As the only available data include numerous uncertainties and all are expressed with a percentage precision. Furthermore a probabilistic approach with valid error models has been proposed by H. Erzberger and R. Paielli. Their method was validated on simulations and experimentation on real traffic.

So, first the conflict detection method of H. Erzberger and R. Paielli will be presented. And a first set of enhancements will be proposed, mainly regarding the modelling of position uncertainty. The tuning of the model by reproducing the results obtained in [1,2] will constitute the initial step of this approach. Then, some model propositions will be done and discussed. Finally a new model of position error will be presented. A simulation under Matlab will be made in order to validate the experimental results.

## 1.5. BACKGROUND

There are different levels of information accuracy regarding the aircraft and navigation [2].

Navigation can be :

- By FMS (Flight Management System) : it is a system that computes a trajectory and provides guidance. It can follow a route with a high accuracy (e.g. 5nmi at 95%).
- By A/P (Auto-Pilot) : it acquires and maintains flight parameters like heading, altitude and velocity. The route accuracy can be given.
- Manual.

Information on the aircraft can be in the form of :

- TCP (Trajectory Change Points) : all route points where the aircraft will change one of its flight parameters, and an estimation of time and velocity at which they would be reached.
- TV (Target values) : flight parameters that the A/P will reach (e.g. if the aircraft starts climbing, the altitude at which it will stop).
- FS (Flight States) : $(x,\vec{v})$ current position and velocity of the aircraft.

The aim is to cross these different levels. Many combinations are obtained. Only typical cases will be treated. Starting with models containing the maximum amount of information and degrading them progressively.

The source of intruder information is required It will be assumed that they are normally given by ADS-B [3]. In addition, it should be noted that the “own” aircraft can have limited information about itself.

With this information, the trajectories of both aircraft can be reconstituted taking uncertainly into account.

If there is no change of intent during the flights then the trajectory precision will be better for the coupled FMS & TCP than for the A/P & TV. The latter is more precise than Manual & FS.

# 2. SIMPLE STRAIGHT FLIGHTS IN 2D

In this part the method used by H. Erzberger and R. Paielli to estimate the probability of conflict in a simple case will be described. First, the method of combining two prediction error co-variances into a single co-variance of the relative position is discussed. Next, a co-ordinate transformation is proposed that transforms the combined error co-variance into a standard form. Then, the analytical solution for the conflict probability in two dimensions is developed.

The design is based on an approach that combines deterministic trajectory prediction and stochastic conflict analysis to achieve reliable conflict detection.

## 2.1. ASSUMPTIONS

The two aircraft fly in straight lines at constant velocities.

The aircraft maintain their cross-track position by using a FMS and communicate their flight parameters with ADS-B. Their trajectories are predicted approximately for the next 20 or 30 minutes.

We do not take the wind into consideration in this part.

Let the separation minima be $s_c$.

Let the initial position of aircraft 1 be $(x_0^1, y_0^1)$ in an Earth-referenced co-ordinate system, and its ground velocity be $\vec{v}_1$.

Let the initial position of aircraft 2 be $(x_0^2, y_0^2)$ in an Earth-referenced co-ordinate system, and its ground velocity be $\vec{v}_2$.

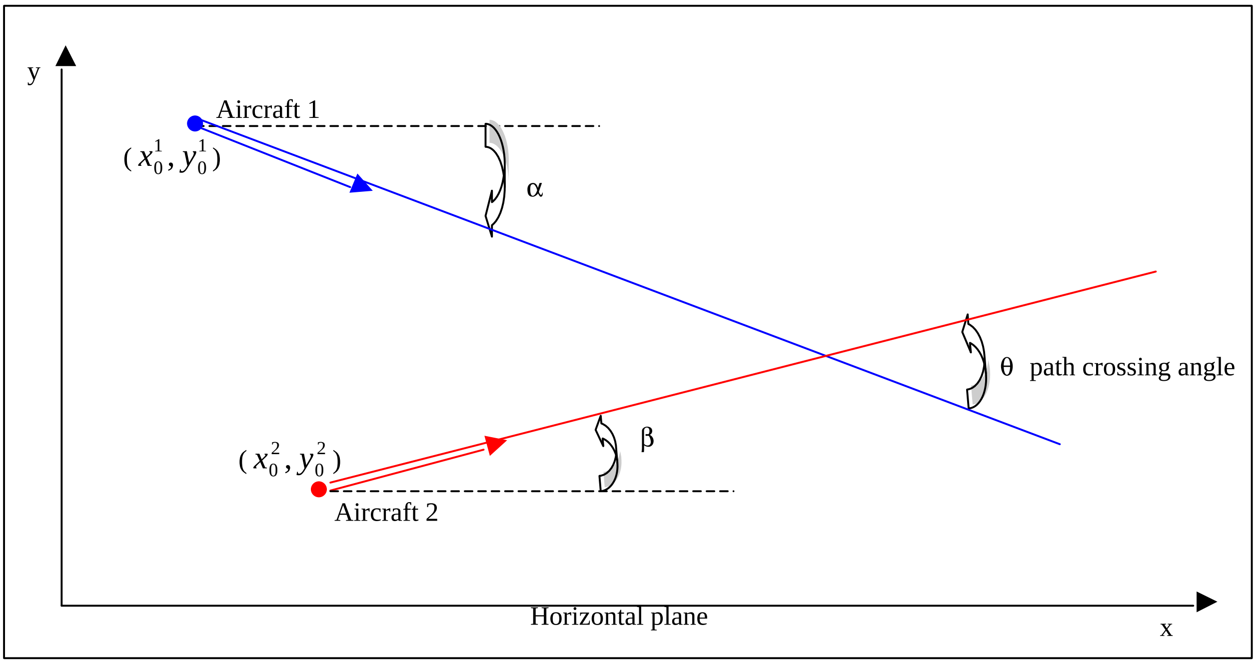


**Figure 3: trajectories of the aircraft in a Earth-fixed reference co-ordinate system**

Let $\alpha$ be the angle between the aircraft 1 velocity vector and the x-axis.

Let $\beta$ be the angle between the aircraft 2 velocity vector and the x-axis.

Let $\theta$ be the path-crossing angle.

Let the horizontal position error of aircraft i be $\sigma_{hpi}$ and its horizontal velocity error be $\sigma_{hvi.}$, i=1, 2.

It is assumed that the angle between the vector airspeed and the aircraft centre line (angle of attack) is always equal to zero. Therefore, the trajectory prediction error for an aircraft will be modelled as normally distributed, with zero mean and with a covariance matrix that has eigenvectors in the along-track and cross-track directions.

## 2.2. COMBINED ERROR CO-VARIANCE

Two errors of positions have to be considered for a pair of aircraft, represented by two co-variance matrices. They can be combined into a single equivalent co-variance matrix. This combined covariance matrix can be assigned to one of the aircraft, referred to as the "stochastic" aircraft, and the other aircraft, referred to as the "referenced" aircraft, can be regarded as having no position uncertainly. Aircraft 1 is chosen as the stochastic and aircraft 2 is chosen as referenced. The “stochastic” aircraft is fixed, only the “referenced” aircraft is moving.

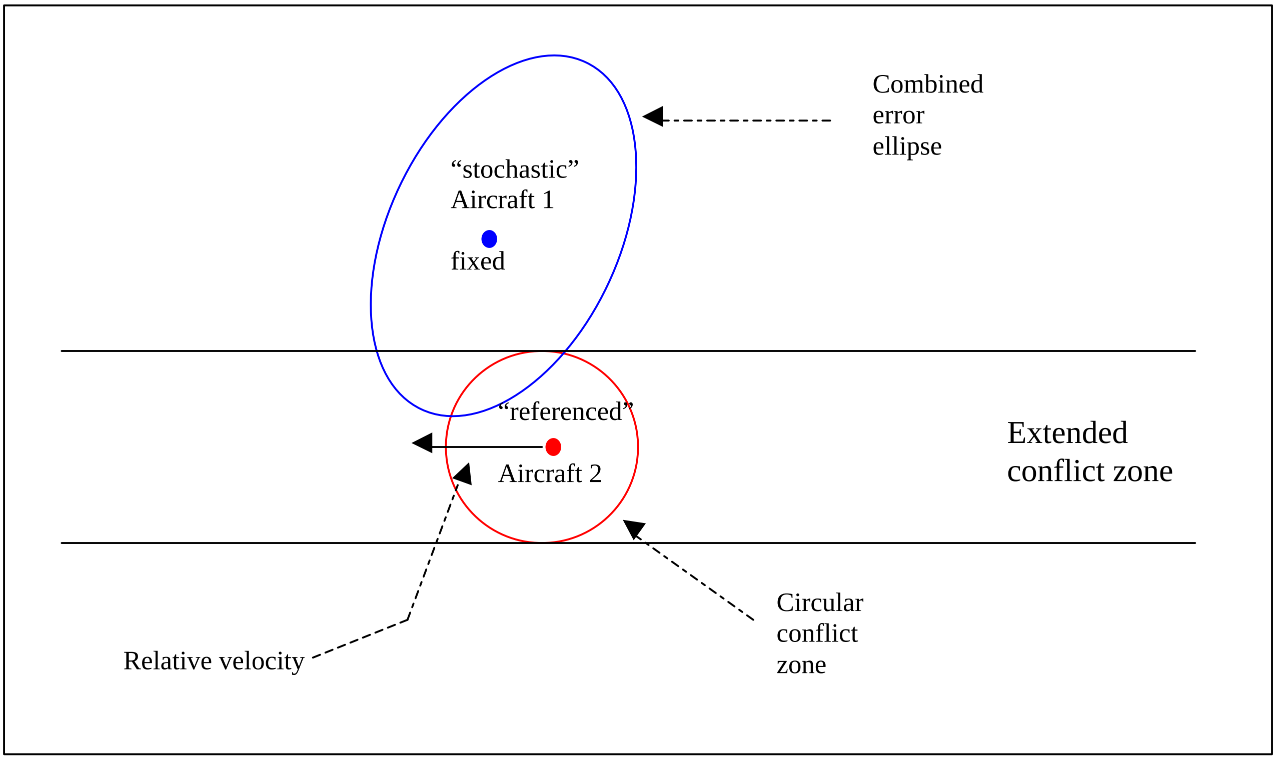


**Figure 4: 2D encounter geometry**

Firstly, the error co-variance matrix of each aircraft is computed. It can be noted that the error ellipses grow with time.

If q is the aircraft position in a co-ordinate system aligned with the aircraft heading, and $\bar{q}$ the corresponding prediction, it represents the corresponding mean :

$\bar{q} = E(q)$

Then $\underline{q}$ is the prediction error : $\underline{q} = q - \bar{q}$

Let S be the corresponding diagonal covariance matrix : $S \equiv cov(\underline{q})$

Where α (respectively β) is the heading angle in the Earth-fixed reference co-ordinate system, then :

$$R^1 = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}, \text{ (respectively } R^2 = \begin{bmatrix} \cos b & -\sin b \\ \sin b & \cos b \end{bmatrix})$$

is the rotation matrix that transforms the heading-aligned co-ordinates to the reference co-ordinate system.

Let $p$, $\bar{p}$, $\underline{p}$, be the real position, the predicted position, and the predicted position error in the reference co-ordinate system respectively.

Then : i=1, 2

$\bar{p} = R^i \bar{q}$

$\underline{p} = R^i \underline{q}$

$Q^i$ º $cov(\underline{p}) = R^i S R^{iT}$

With $Q^i$ the individual covariance matrix in the reference co-ordinate system.

There are different ways to view the errors in velocity. It can be seen as a constant error that linearly grows with time or as a constant error on average, with “large” variations.

So here it is chosen to model the position error resulting from velocity as a Brownian process. For information, the covariance matrix for a constant error is also given.

According to H. Erzberger, the cross-track rms. (root mean square) error is constant and the along track rms. error starts at zero and grows linearly with time. So the individual co-variance matrix at time t, is in the following form :

$$Q^i(t) = R^i \begin{bmatrix} t^2 (s_{hvi})^2 & 0 \\ 0 & (s_{hpi})^2 \end{bmatrix} R^{iT}$$

In this model, it is considered that instruments make a constant error in position as well as velocity error dependant on time.

With W a Brownian and t fixed :

$$dx = vdt + s_{hv} d W_v$$
$$x(t) = v(t - t_o) + s_{hv} W(t - t_0)$$
$$\operatorname{var}(x(t)) = \operatorname{var}(x(t_0)) + \operatorname{var}(s_{hv} W(t - t_0))$$
$$\operatorname{var}(x(t)) = s_{hp}^2 + (t - t_o) s_{hv}^2$$

Therefore it is obtained at time t :

$$Q^1(t) = R^1 \begin{bmatrix} (s_{hp1})^2 + t(s_{hv1})^2 & 0 \\ 0 & (s_{hp1})^2 \end{bmatrix} R^{1T}$$

$$Q^2(t) = R^2 \begin{bmatrix} (s_{hp2})^2 + t(s_{hv2})^2 & 0 \\ 0 & (s_{hp2})^2 \end{bmatrix} R^{2T}$$

Now, the combined co-variance matrix will be computed.

The position difference is : $\Delta p$ º $p_1 - p_2$

The predicted difference of that position difference is : $\Delta \bar{p}$ º $\bar{p}_1 - \bar{p}_2$

And the prediction error is : $\Delta \underline{p}$ º $\underline{p}_1 - \underline{p}_2$

The combined prediction co-variance error is : $M$ º $cov(\Delta \underline{p}) = Q^1 + Q^2 - Q^{12}$

Where the cross-correlation term $Q^{12}$ is defined as : $Q^{12}$ º $E(\underline{p}_1 \underline{p}_2^T + \underline{p}_2 \underline{p}_1^T)$

It represents the effects of wind correlation and also the correlated instrument errors. It is chosen here not to consider the correlation of the instrument errors, because they are small and because it simplifies the computation. Moreover, no attention is given to the wind in this part, so the $Q^{12}$ value is zero.

Thus the combined co-variance matrix is :

$$M = R^1 \begin{bmatrix} (s_{hp1})^2 + t(s_{hv1})^2 & 0 \\ 0 & (s_{hp1})^2 \end{bmatrix} R^{1T} + R^2 \begin{bmatrix} (s_{hp2})^2 + t(s_{hv2})^2 & 0 \\ 0 & (s_{hp2})^2 \end{bmatrix} R^{2T}$$

M can be classified as a symmetric definite positive square matrix.

## 2.3. CO-ORDINATE TRANSFORMATION

A transformation of the combined error ellipse into a unit circle is required, with the aim of simplifying the conflict probability computation.

Let $r$ represent the transformed co-ordinates position.

A general co-ordinate transformation is of the form : $r = Tp$

Where $T$ is a transformation matrix to be determined.

The transformation for velocity and other vectors are in the same form. So :

$Dr = r_1 - r_2$

$\Delta \overline{r} = \overline{r}_1 - \overline{r}_2$

In the transformed co-ordinate system, the mean prediction error is still zero, and the combined error covariance is : $cov(\Delta \underline{r})$ ° $TMT^T$

To transform the combined covariance into the form of a unit circle, it is sufficient to find T as : $TMT^T = I$

A Cholesky decomposition of $M$ is of the form : $M = LL^T$, where $L$ is lower triangular with positive diagonal elements.

If $T$ is of the form : $T = RL^{-1}$

Then : $cov(\Delta \underline{r}) = I$

The combined error ellipse is therefore in the standard form of a unit circle. The conflict boundary, which was a circle in the original co-ordinate system, is an ellipse in the transformed co-ordinate system.

The rotation matrix $R$ has been chosen such that the relative velocity is in the positive or negative $x$-direction.

If : $\Delta v$ ° $v_1 - v_2$

is the relative velocity in the original co-ordinate system, and

$\Delta\gamma$ ° $L^{-1}\Delta v$ ° $\begin{bmatrix} \Delta g_x \\ \Delta g_y \end{bmatrix}$

is the partially transformed relative velocity, then

$$R = \frac{1}{\|\Delta g\|} \begin{bmatrix} \Delta g_x & \Delta g_y \\ -\Delta g_y & \Delta g_x \end{bmatrix}$$

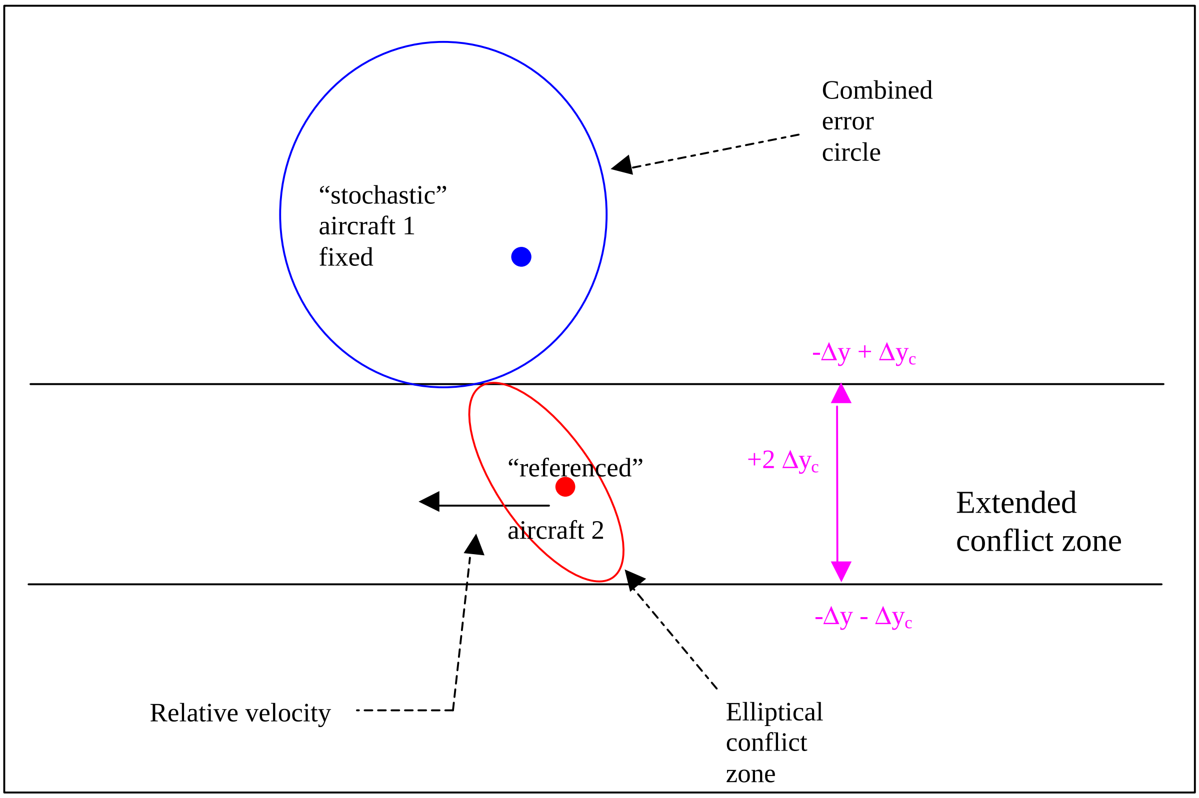


**Figure 5: Transformed 2D-encounter geometry**

## 2.4. CONFLICT PROBABILITY

Having the error ellipsoid in the form of a unit circle simplifies the probability computation considerably because the corresponding 2D probability density function can be represented as a radial surface over the error circle, where total volume under the surface is unity.

So, the probability density function can be expressed as the product of two identical one-dimensional functions :

$p(x, y) = p(x)p(y)$, where $p(x) = exp(-x^2/2)/\sqrt{2p}$ .

The individual error ellipses grow with time, as so does the combined error ellipse. To find the conflict probability, it is required to have constant trajectory prediction errors during the encounter or period of potential conflict. It is chosen to approximate the combined error ellipse with the ellipse at the CPA (t=tCPA). An ellipse with a fixed size that moves with a velocity equal to the difference between the two aircraft velocities (i.e. the relative velocity) is obtained.

This approximation is valid when the period of potential conflict is short, that is to say when the aircraft have no parallel flight paths.

LEMMA 1 :

> For constant velocities, the closest time of approach is :
>
> tCPA = $t_0 - \frac{\Delta p_0^T \Delta v}{\Delta v^T \Delta v}$
>
> where $\Delta p_0$ is the position difference at time $t_0$, and $\Delta v$ different from zero, is the constant velocity difference.

*Proof : appendix A*

The position difference at minimum separation is then : $\Delta p_m = \Delta p_0 + (t_m - t_0)\Delta v$

The minimum separation distance (dCPA) is $\|\Delta p_m\|$.

A conflict occurs when the dCPA is lower than the minimum allowed distance between the aircraft ($s_c$).

The probability of a conflict in a particular time t is the portion of volume that is within the elliptical conflict zone ε(t) :

$$P_c(t) = \iint_{e(t)} p(x,y)dxdy$$

But, this probability is not as important as the total probability of conflict for the encounter, which can be determined as follows. The elliptical zone is projected along a line parallel to the relative velocity to form an extended conflict zone.

Then the conflict probability is equal to the portion of the volume under the probability density surface that is within this extended conflict zone. (Justification 1 in appendix A).

To integrate the density function, the boundaries of the extended conflict zone must be computed. The boundaries of the extended zone are the minimum and the maximum values of y on the elliptical conflict boundary.

Let $\Delta p_c$ and $\Delta r_c$ represent the original and the transformed co-ordinates, respectively, of points on the conflict boundary relative to the reference aircraft.

The equation of the conflict boundary is : $\|\Delta p_c\| = \|W\Delta r_c\| = s_c$

Where $W = T^{-1}$

This equation can be squared, then expanded according to :

$$\Delta r_c \equiv \begin{bmatrix} \Delta x_c \\ \Delta y_c \end{bmatrix},\ W^T W \equiv \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

The resulting equation for the elliptical conflict boundary is: $a\Delta x_c^2 + 2b\Delta x_c \Delta y_c + c\Delta y_c^2 = s_c^2$

LEMMA 2 :

The minimum and maximum values of $\Delta y_c$ can then be determined setting the discriminant of that quadratic equation to zero and solving for $\Delta y_c$. The result is:

$\Delta y_c = \pm s_c \sqrt{a/(ac - b^2)}$

*proof : appendix A*

Note that $a$ is positive and $(ac - b^2)$ is positive and invariant with respect to rotation for any ellipse, so the argument of the square root function must also be positive.

The expression of the conflict probability $P_c$ can be simplified as follow :

$$P_c = \int_{-Dy-Dy_C}^{-Dy+Dy_C} \int_{-\infty}^{+\infty} p(x,y)dxdy = \int_{-Dy-Dy_C}^{-Dy+Dy_C} p(y)dy \int_{-\infty}^{+\infty} p(x)dx = \int_{-Dy-Dy_C}^{-Dy+Dy_C} p(y)dy$$

$$P_c = P(-Dy + Dy_c) - P(-Dy - Dy_c)$$

where $\Delta y = y_1 - y_2$ is the $y$-co-ordinate of the stochastic aircraft with respect to the reference aircraft, and P is the cumulative normal probability function (cf. figure 5).

## 2.5. COMMENT

A method to estimate the conflict probability for a pair of aircraft flying in straight lines, in 2D, at constant velocities and without taking wind effects into account, has been shown. This is a restricted case.

Now, it is useful to investigate what happens when the aircraft fly with winds, change of velocities, headings and altitudes. On the one hand, a natural way to go in this study might be to calculate position error estimations at fixed times. That is to say, that all positions are uncertain and therefore, the aircraft turn after fixed times. But on the other hand, that does not fit well with the assumptions of navigation. Because the FMS ensures that the aircraft turns exactly at the specified points (modulus the instrument errors).

Hence, the errors are translated in terms of delay from the time of passing over required positions. The time is considered as a random variable. Then, the difficulty is to identify a conflict when it occurs. How can the separation distance be considered with this new model?

A lot of things about the time as a random variable are still unknown. So, it is vital to try to find, if it is possible, an approximate mathematical description of its behaviour. The beginning of this next study can be found in appendix B.

# 3. THE WIND EFFECTS

In this section an explanation of how the wind has an impact on aircraft trajectories will be given It will help in the comprehension and calculation of the combined covariance cross-correlation term resulting from the uncompensated effects of winds.

Let $\vec{e}_1$ be the normalised direction vector of aircraft 1 and $\vec{e}_2$ be the normalised direction vector of aircraft 2.

The wind can be decomposed into a mean vector plus a noise component: $wind = \vec{w} + \Delta\vec{w}$

The effects of the average wind can be determined, but it is impossible to anticipate the noise and therefore to compensate for it. So a focus on the wind noise will be made here.

Let the noise on the wind be $\Delta\vec{w} = \begin{pmatrix} \Delta w_1 \\ \Delta w_2 \end{pmatrix}$ and the resulting effect on position be $\partial\vec{w} = \begin{pmatrix} \partial w_1 \\ \partial w_2 \end{pmatrix}$.

Let the difference of position resulting from winds on the aircraft j be $\hat{p}_{wj}$ . Therefore :

$$\hat{p}_{w1} = x_1 \partial\vec{w}.\vec{e}_1.\vec{e}_1 = x_1 \begin{pmatrix} \partial w_1 \\ \partial w_2 \end{pmatrix} \vec{e}_1.\vec{e}_1 \text{ and } \vec{e}_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\hat{p}_{w2} = x_2 \partial\vec{w}.\vec{e}_2.\vec{e}_2 = x_2 \begin{pmatrix} \partial w_1 \\ \partial w_2 \end{pmatrix} \vec{e}_2.\vec{e}_2 \text{ and } \vec{e}_2 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

with $x_j$ a coefficient particular to the aircraft j.

As seen before (section 2.2), the cross-correlation term represents the wind effects correlation. Then :

$$E(\underline{p}_1 \underline{p}_2^T + \underline{p}_2 \underline{p}_1^T) = E\left(\hat{p}_{w1}\hat{p}_{w2}^T + \hat{p}_{w2}\hat{p}_{w1}^T\right)$$

with :

$$E\left(\hat{p}_{w1}\hat{p}_{w2}^T\right) = E\left[x_1 \begin{pmatrix} \partial w_1 \\ \partial w_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \vec{e}_1 \vec{e}_2^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}^T \begin{pmatrix} \partial w_1 \\ \partial w_2 \end{pmatrix}^T x_2\right]$$

$$= \vec{e}_1 \vec{e}_2^T x_1 x_2 \left(x_1 y_1 \partial w_1^2 + (x_1 y_2 + x_2 y_1)\partial w_1 \partial w_2 + x_2 y_2 \partial w_2^2\right)$$

And at a fixed small interval of time $\partial t$:

$$E(\underline{p}_1 \underline{p}_2^T + \underline{p}_2 \underline{p}_1^T) = x_1 x_2 \left(\vec{e}_1 \vec{e}_2^T + \vec{e}_2 \vec{e}_1^T\right)\left(x_1 y_1 \partial w_1^2 + \left(x_1 y_2 + x_2 y_1\right)\partial w_1 \partial w_2 + x_2 y_2 \partial w_2^2\right)$$

Integration, produces :

$$E(\underline{p}_1 \underline{p}_2^T + \underline{p}_2 \underline{p}_1^T) =$$
$$x_1 x_2 \left(\vec{e}_1 \vec{e}_2^T + \vec{e}_2 \vec{e}_1^T\right)\left(x_1 y_1 \operatorname{var}^2(\partial w_1) + \left(x_1 y_2 + x_2 y_1\right)\operatorname{cov}(\partial w_1, \partial w_2) + x_2 y_2 \operatorname{var}^2(\partial w2)\right)$$

So the cross-correlation between the two positions errors resulting from winds, can be decomposed into three parts, where two of the parts are independent from each other.

# 4. More realistic modelling of flights in 2D: Turns are instantaneous at fixed times

The aircraft fly with winds, change of velocities and headings.

## 4.1. Assumptions

The two aircraft fly in the same horizontal plane. Their trajectories are predicted typically for the next 20 or 30 minutes ahead. The aircraft are assumed to fly using a FMS and communicate using ADS-B. So, they maintain their track and broadcast their TCP and Navigation Uncertainly Categories (NUC).

It is assumed that the NUC can take into account, for all aircraft, the errors resulting from winds by dividing the airspace into areas of weather uncertainty. In each area, the mean wind direction is known. Giving the variance of position errors resulting from winds.

It is assumed for simplicity that the changes of velocity are instantaneous.

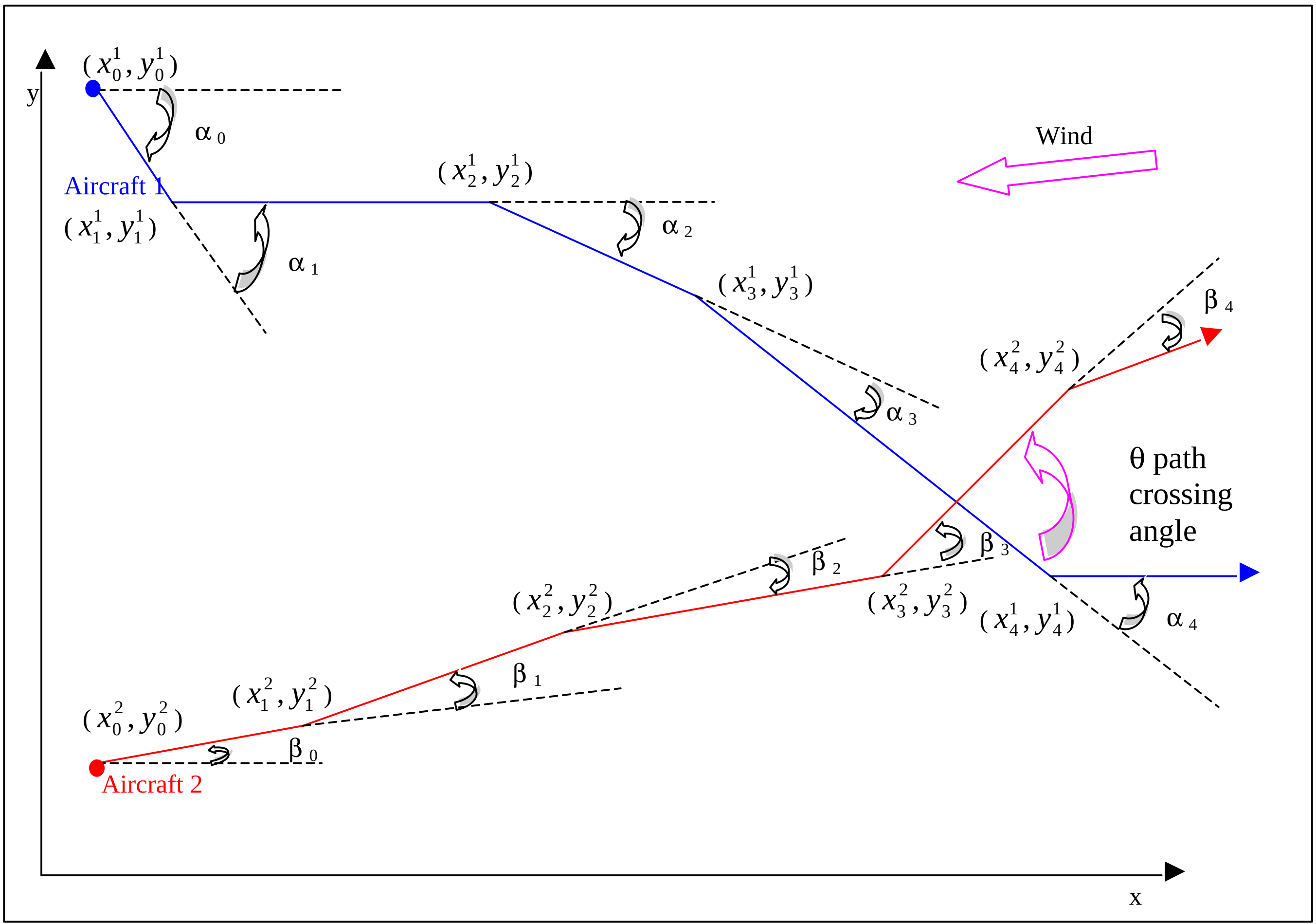


**Figure 5 : trajectories of the aircraft in the Earth reference co-ordinate system in a horizontal plane**

Let the position of aircraft 1 at i-th TCP be $(x_i^1, y_i^1)$ in the Earth reference co-ordinate system and its velocity be $\vec{v}_i^1$.

Let the position of aircraft 2 at i-th TCP be $(x_i^2, y_i^2)$ in the Earth reference co-ordinate system and its velocity be $\vec{v}_i^2$.

Let the horizontal position error of aircraft 1 be $s_{hp1}$, its horizontal velocity error be $s_{hv1}$ and its wind resulting error be $s_{hw1}^{i}$ for the segment [$TCP_i$, $TCP_{i+1}$]. Let the horizontal position error of aircraft 2 be $s_{hp2}$, its horizontal velocity error be $s_{hv2}$ and its wind resulting error for the segment [$TCP_i$, $TCP_{i+1}$] be $s_{hw2}^{i}$.

As before, the trajectory prediction error for an aircraft will be modelled as normally distributed, with zero mean and with a covariance matrix that has eigenvectors in the along-track and cross-track directions.

Let $\alpha_i$ and $\beta_i$ be, respectively the changing of heading for aircraft one and two, at $TCP_i$ , with the first heading estimated in a fixed Earth co-ordinate system.

A simple assumption, is that turns can be instantaneous at given times. The prediction errors are always ellipses with principal axes in the along-track and cross-track directions.

For each aircraft new trajectories points are calculated, with the aim of having the same time division. That is to say, the predicted states of an aircraft when the other fly over a TCP, are needed. Then, for each period between two successive TCPs, the probability of conflict is calculated and the method of H.Erzberger and R. Paielli is adapted.

## 4.2. COMBINED ERROR COVARIANCE

As in section 2, we can combine the two errors into a single error and assign it to one of the aircraft.

Let $t_i$ be the estimated time when the aircraft flies over $TCP_i$.

Let $Q^j(t)$ be the individual covariance matrix for the aircraft j, in an Earth-fixed reference co-ordinate system.

Let $R_i^j(t)$ be the corresponding rotation matrix that transforms the heading-aligned co-ordinates to the Earth-fixed reference co-ordinate system on a segment [TCPi; TCPi+1].

The time $t_i$ is variable but, first it is assumed that the aircraft change their flight parameters at fixed times.

If $t$ Î $[t_i ; t_{i+1}[$ then :

$$Q^j(t) = s_{hpj}^2 I + \sum_{k=0}^{i-1} (t_{k+1} - t_k) A_k^j + (t - t_i) A_i^j$$

$$A_k^j = R_k \left( \left( s_{hvj}^{\ k} \right)^2 + ( s_{hwj}^{\ k} )^2 \right) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} R_k^T$$

with :

$$R_i^1(t) = \begin{bmatrix} \cos a_i & -\sin a_i \\ \sin a_i & \cos a_i \end{bmatrix}$$

$$R_i^2(t) = \begin{bmatrix} \cos b_i & -\sin b_i \\ \sin b_i & \cos b_i \end{bmatrix}$$

Therefore the combined prediction covariance matrix is :

$$M \equiv cov(\Delta \underline{p}) = Q^1 + Q^2 - Q^{12}$$

Where the cross-correlation term $Q^{12}$ is defined as :

$$Q^{12} \equiv E(\underline{p}_1 \underline{p}_2^T + \underline{p}_2 \underline{p}_1^T)$$

This cross-correlation term represents only the correlated effects of winds (as explained in section 2). If the two aircraft are far from each other, it can be considered that $Q^{12}$ is negligible. Otherwise, it is necessary to calculate it.

LEMMA 3 :

> $E(\underline{p}_1 \underline{p}_2^T + \underline{p}_2 \underline{p}_1^T) = \pm (\vec{e}_2 \vec{e}_1^{\,T} + \vec{e}_1 \vec{e}_2^{\,T}) s_{hw1} s_{hw2} t$
>
> with $\vec{e}_1$ the normalised direction vector of aircraft 1 and $\vec{e}_2$ the direction vector of aircraft 2.

*proof : Appendix A*

Therefore the total encounter matrix correlation is :

$$Q^{12} = \sum_{k=0}^{i-1} \pm \left( (\vec{e}_2^{\,k} \vec{e}_1^{\,kT} + \vec{e}_1^{\,k} \vec{e}_2^{\,kT}) s_{hw1}^{\ k} s_{hw2}^{\ k} (t_{k+1} - t_k) \right) \pm (\vec{e}_2^{\,i} \vec{e}_1^{\,iT} + \vec{e}_1^{\,i} \vec{e}_2^{\,iT}) s_{hw1}^{\ i} s_{hw2}^{\ i} (t - t_i)$$

For simplicity, it is assumed that there is no wind correlation when the aircraft are in different weather areas.

(For a more detailed study of the influence of winds see section 3).

## 4.3. CO-ORDINATE TRANSFORMATION & CONFLICT PROBABILITY

As in section 2, almost the same co-ordinate transformation and probability calculation is made. Thus, the probability density function can be de-coupled into the product of two identical one-dimensional functions :

*p(x, y) = p(x)p(y)*, where $p(x) = exp(-x^2/2)/\sqrt{2p}$ .

LEMMA 4 :

For constant velocities on the path segments and for $\Delta v$ not equal to zero, the time at which the minimum predicted separation occurs is :

$$\text{tCPA} = t_m = \begin{cases} (t_{i+1}^-) \text{ if } -\frac{\Delta p_i^T \Delta v}{\Delta v^T \Delta v} \geq t_{i+1} - t_i \\ (t_i - \frac{\Delta p_i^T \Delta v}{\Delta v^T \Delta v}) \text{ if } t_{i+1} - t_i > -\frac{\Delta p_0^T \Delta v}{\Delta v^T \Delta v} > 0 \\ (t_i) \text{ if } -\frac{\Delta p_i^T \Delta v}{\Delta v^T \Delta v} \leq 0 \end{cases}$$

where $\Delta p_i$ is the position difference at time $t_i$, and $\Delta v$ is the constant velocity difference.

*Proof : Appendix A*

The position difference at minimum separation is then : $\Delta p_m = \Delta p_i + (t_m - t_i)\Delta v$

If $\Delta v$ is equal to zero, then the position difference is constant.

The minimum separation distance (dCPA) is $\|\Delta p_m\|$.

The minimum and maximum values of $\Delta x_c$ and $\Delta y_c$ can then be determined as explained in section 2. The results are :

$$\Delta x_c = \pm s_c \sqrt{\frac{c}{ac - b^2}}$$

$$\Delta y_c = \pm s_c \sqrt{a/(ac - b^2)}$$

The probability of a conflict in a particular time is the portion of volume that is within the elliptical conflict zone ε(t). So :

$$\forall t, P_c(t) = \iint_{e(t)} p(x, y)dxdy$$

But, this probability is not as important as the total probability of conflict for the encounter, which can be determined as follows. For each segment [$TCP_i$;$TCP_{i+1}$], the elliptical conflict zone can be projected along a line parallel to the relative velocity. Then, it can be bounded with the maximum and minimum value of $\Delta x_c$ (for $t \in [t_i\ ;\ t_{i+1}\ -1]$) to obtain a rectangle.

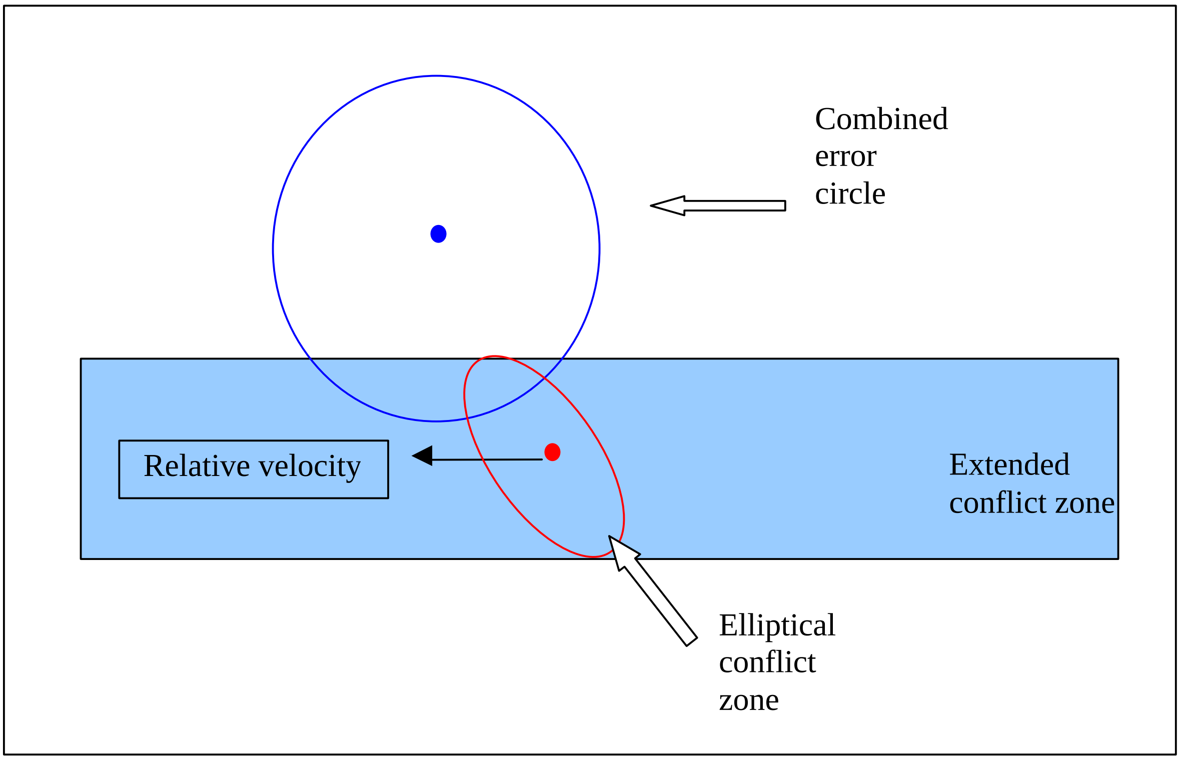


**Figure 6: Transformed 2D encounter geometry for t Î [$t_i$;t $_{i+1}$-1[**

The conflict probability on this segment is :

$$P_c(t \in [t_i;t_{i+1}[) = \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} \int_{-\Delta x-\Delta x'}^{-\Delta x+\Delta x''} p(x,y)dxdy = \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} p(y)dy \int_{-\Delta x-\Delta x'}^{-\Delta x+\Delta x''} p(x)dx$$

$$P_c(t \in [t_i;t_{i+1}[) = P(-\Delta y + \Delta y_c)P(-\Delta x + \Delta x'') - P(-\Delta y - \Delta y_c)P(-\Delta x - \Delta x')$$

where :

- $\Delta x = x_1 - x_2$ , $\Delta y = y_1 - y_2$ are respectively the x and *y*-co-ordinate of the stochastic aircraft with respect to the reference aircraft.
- P is the cumulative normal probability function.
- $$\Delta x' = (s_c\sqrt{c/ac - b^2}) + (t_m - t_i)\Delta v_i$$
- $$\Delta x'' = \left(s_c\sqrt{c/ac - b^2}\right) + (t_{i+1} - t_m)\Delta v_i$$

Therefore, the total conflict probability can be bounded as follows :

$$\max_i P_c(t \in [t_i;t_{i+1}[) < P_c \leq \sum_i P_c(t \in [t_i;t_{i+1}[)$$

Similarly the worst case conflict probability on a segment is given by:

$$P_{worst} = \max_i P_c(t \in [t_i;t_{i+1}[)$$

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# 5. SIMPLE STRAIGHT FLIGHTS IN 3D

The aircraft fly in straight lines with constant velocities and altitude change rates.

## 5.1. ASSUMPTIONS AND COMBINED COVARIANCE

If two aircraft are in level flight at different altitudes or if one or both of the aircraft are climbing or descending, then the problem is three-dimensional.

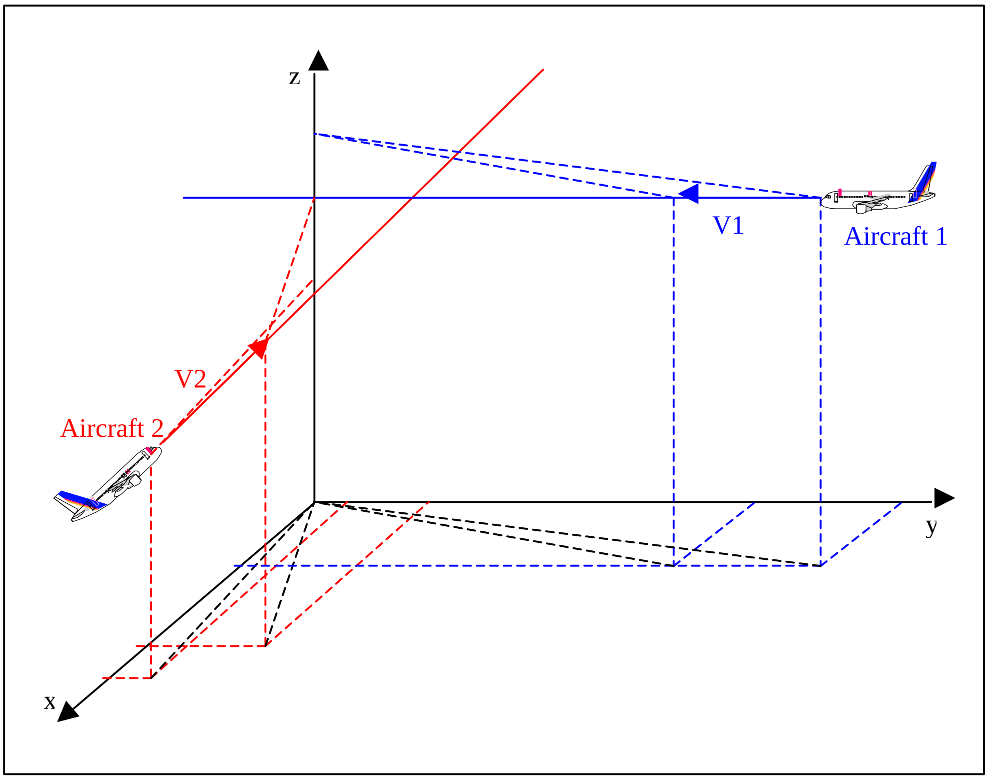


**Figure 7 : example of a three dimensional case**

The three dimensional case is more difficult to illustrate than the two-dimensional case, but it is similar in principal.

The wind is not taken into consideration in this part (i.e. heading and track angle are assumed to be equal). The two aircraft fly in straight lines. Their trajectories are typically predicted for the next 20 or 30 minutes. It is assumed that the aircraft fly at constant velocities and maintain their cross-track position using a FMS.

As mentioned before, the combined error ellipsoid is centred on the stochastic aircraft, and the ellipsoidal conflict zone is centred on the reference aircraft. Here, the conflict zone is not a sphere because the vertical safety distance is not equal to the horizontal one. The real conflict zone is a cylinder with an elliptical cross-section, but for simplicity it is assumed to be an ellipsoid.

Let the horizontal separation minima be $s_{hc}$, and the vertical separation minima be $s_{vc}$.

Let the initial position of aircraft 1 be $(x_0^1, y_0^1, z_0^1)$, and its velocity be $\vec{v}_1$.

Let the initial position of aircraft 2 be $(x_0^2, y_0^2, z_0^2)$, and its velocity be $\vec{v}_2$.

Let the horizontal position error of aircraft 1 be $\sigma_{hp1}$ and its vertical position error be $\sigma_{vp1}$.

Let its horizontal velocity error be $\sigma_{hv1,}$ and its vertical velocity error be $\sigma_{vv1}$.

Let the horizontal position error of aircraft 2 be $\sigma_{hp2}$ and its vertical position error be $\sigma_{vp2}$.

Let its horizontal velocity error be $\sigma_{hv2,}$ and its vertical velocity error be $\sigma_{vv2}$.

It is assumed that the angle between the vector airspeed and the aircraft centre line (angle of attack) is always equal to zero. Therefore, the trajectory prediction error for an aircraft will be modelled as normally distributed, with zero mean and with a covariance matrix that has eigenvectors in the along-track, cross-track and vertical directions.

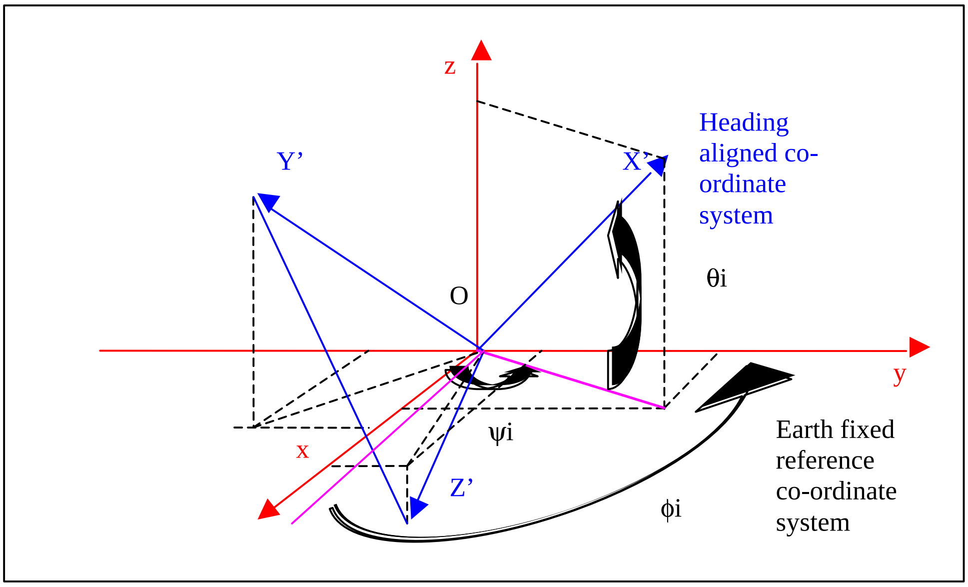


**Figure 8: the heading aligned co-ordinate system and an Earth-fixed reference co-ordinate system**

Let $\theta_j$ be the angle between the Earth reference co-ordinate system x-axis and the projection of the Heading aligned co-ordinate system x'-axis on plan (Ox,Oy).

Let $\psi_j$ be the angle between the projection of the Heading aligned co-ordinate system x'-axe on plan (Ox,Oy) and the Heading aligned co-ordinate system x'-axis.

Let $\phi_i$ be the angle between the intersection of planes (Oy',Oz') and (Ox,Oy), and the Earth reference co-ordinate system y-axis.

$$R^j = \begin{bmatrix} \cos y^j \cos q^j & -\sin y^j \cos f^j + \cos y^j \sin q^j \sin f^j & \cos y^j \sin q^j \cos f^j + \sin y^j \sin f^j \\ \sin y^j \sin q^j & \cos y^j \cos f^j + \sin y^j \sin q^j \sin f^j & -\cos y^j \sin f^j + \sin y^j \sin q^j \cos f^j \\ -\sin q^j & \cos q^j \sin f^j & \cos q^j \cos f^j \end{bmatrix}$$

Because the wind is not taken into consideration, the angle $\phi$ is negligible. So $R^j$ can be written as follows :

$$R^j = \begin{bmatrix} \cos\psi^j \cos\theta^j & -\sin\psi^j & \cos\psi^j \sin\theta^j \\ \sin\psi^j \sin\theta^j & \cos\psi^j & \sin\psi^j \sin\theta^j \\ -\sin\theta^j & 0 & \cos\theta^j \end{bmatrix}$$

$$Q^j(t) = R^j \begin{bmatrix} (\sigma_{hpj})^2 + t(\sigma_{hvj})^2 & 0 & 0 \\ 0 & (\sigma_{hpj})^2 & 0 \\ 0 & 0 & (\sigma_{vpj})^2 + t(\sigma_{vvj})^2 \end{bmatrix} R^{jT}$$

with *j*=1,2

Therefore the combined prediction covariance matrix is : $M = Q^1 + Q^2$

There is no cross-correlation term because there is no wind (and because as before the correlation of instrument errors is not considered).

## 5.2. CO-ORDINATE TRANSFORMATION

The co-ordinate system is changed to transform the error covariance ellipsoid into a unit sphere.

Let $r$ represent the transformed co-ordinates position.

A general co-ordinate transformation is of the form : $r = Tp$

Where *T* is a transformation matrix to be determined.

The transformation for velocity and other vectors is in the same form. So :

$\Delta r = r_1 - r_2$

$\Delta \bar{r} = \bar{r}_1 - \bar{r}_2$

In the transformed co-ordinate system, the mean prediction error is still zero, and the combined error covariance is : $cov(\Delta \underline{r}) \equiv TMT^T$

A Cholesky decomposition of *M* is of the form : $M = LL^T$, where *L* is in lower triangular form, with positive diagonal elements.

If *T* is of the form : $T = RL^{-1}$

Then : $cov(\Delta \underline{r}) = I$

The combined error ellipsoid is therefore in the standard form of a unit sphere. The conflict boundary, which was an ellipsoid in the original co-ordinate system, is still an ellipsoid in the transformed co-ordinate system.

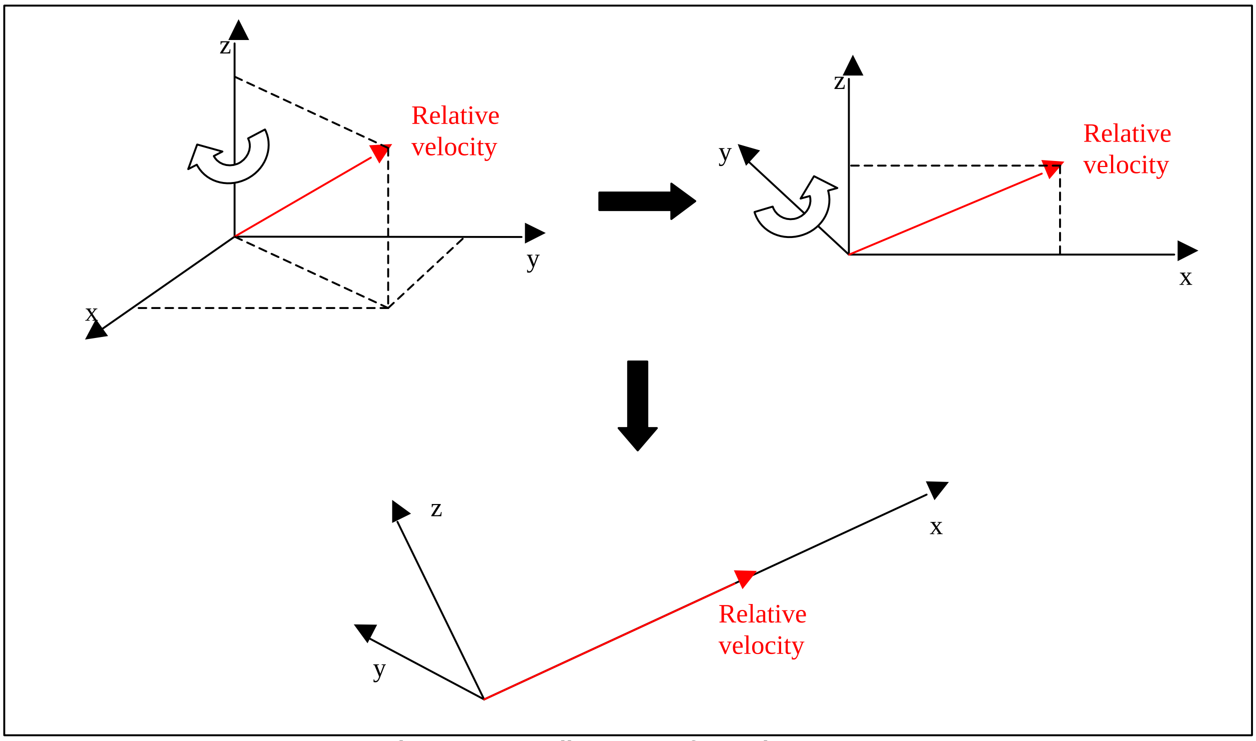


**Figure 9: co-ordinate transformation**

The transformation can still be selected such that the relative velocity is in the positive or negative x-direction.

In order to do this, a rotation in two phases is used : first, a rotation around the z-axis so that the relative velocity is in the x-z plane ; secondly, a rotation around the y-axis so that the relative velocity is in the negative or positive x-direction. (C.f. figure 10).

If $\Delta v \equiv v_1 - v_2$ is the relative velocity in the original co-ordinate system, and

$\Delta\gamma \equiv L^{-1}\Delta v \equiv \begin{bmatrix} \Delta g_x \\ \Delta g_y \\ \Delta g_z \end{bmatrix}$ is the partially transformed relative velocity, then :

$$R = \frac{1}{\|\Delta g\|}\begin{bmatrix} \Delta g_x & \Delta g_y & 0 \\ -\Delta g_y & \Delta g_x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{\Delta g_x^{\,2} + \Delta g_y^{\,2}} & 0 & \Delta g_z \\ 0 & 1 & 0 \\ -\Delta g_z & 0 & \sqrt{\Delta g_x^{\,2} + \Delta g_y^{\,2}} \end{bmatrix}$$

$$R = \frac{1}{\|\Delta g\|}\begin{bmatrix} \Delta g_x\sqrt{\Delta g_x^{\,2} + \Delta g_y^{\,2}} & \Delta g_y & \Delta g_x \Delta g_z \\ -\Delta g_y\sqrt{\Delta g_x^{\,2} + \Delta g_y^{\,2}} & \Delta g_x & -\Delta g_y \Delta g_z \\ -\Delta g_z & 0 & \sqrt{\Delta g_x^{\,2} + \Delta g_y^{\,2}} \end{bmatrix}$$

## 5.3. CONFLICT PROBABILITY

As in the previous case, the conflict zone is extended to study the total probability of conflict for the encounter. This can be determined as follows. The ellipsoid is projected along a line parallel to the relative velocity to form a cylinder. (C.f. figure 6).

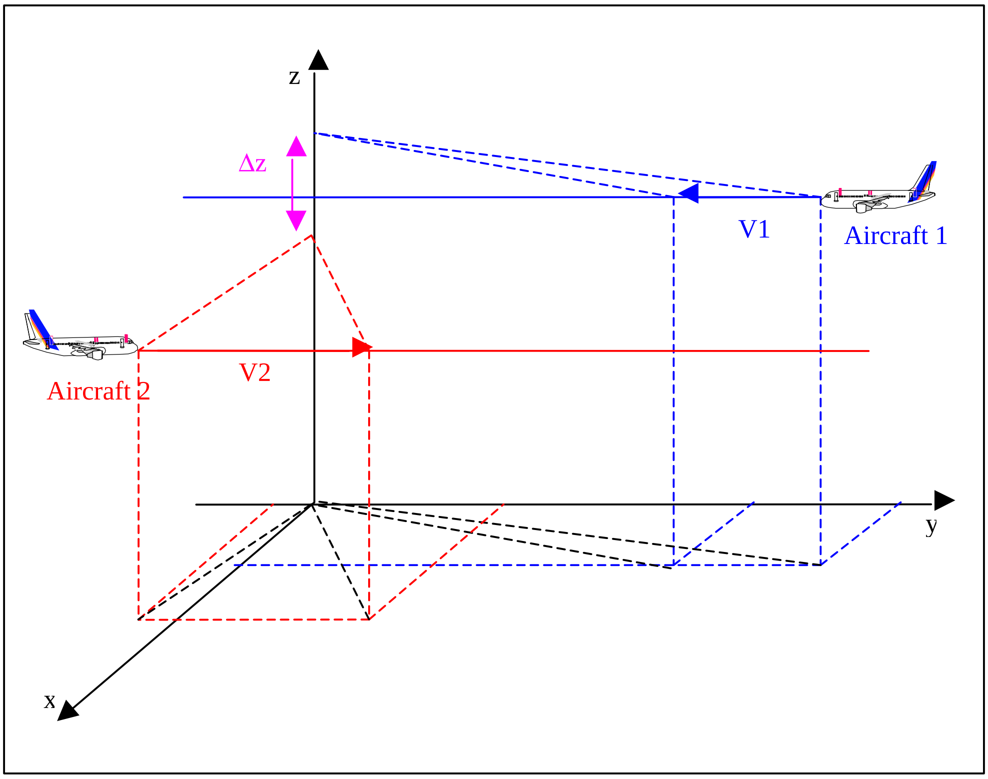


**Figure 10: Example of level flight encounter**

It is assumed that the angle between the airspeed vector and the aircraft centre line (angle of attack) is always equal to zero. Then, the probability density function can be de-coupled producing three identical one-dimensional functions and be represented as a radially symmetric probability distribution, where the total probability is unity.

For the important case of level flight (both aircraft are in level flight, but at different attitudes, e.g. in figure 11), the cylindrical conflict zone appears as a rectangle when viewed horizontally from the direction of the relative velocity. Then the conflict probability can be de-coupled into the product of vertical and horizontal conflict probabilities and can be determined exactly by the analytical methods shown previously, see section 2.4.

So, as seen before, it is necessary to have constant trajectory prediction errors during the period of poential conflict. For this reason, the combined error ellipsoid is approximated by the ellipsoid at tCPA.

This approximation is unimportant when the period of potential conflict is short, that is to say, when the aircraft have no parallel fligth paths.

LEMMA 5 :

For constant velocities, the time at which the minimum predicted separation occurs is :

tCPA = $t_0 - \dfrac{\Delta p_0^T \Delta v}{\Delta v^T \Delta v}$

where $\Delta p_0$ is the position difference at time $t_0$, and $\Delta v$ is the constant velocity difference, (non–zero).

*Proof : c.f. proof lemma 1 in Appendix A* □

The position difference at minimum separation is then : $\Delta p_m = \Delta p_0 + (t_m - t_0)\Delta v$

Now a study of the conflict boundary is required.

Let $U$ be the matrix that transforms the ellipsoid of safety in a unit sphere.

$$U = \begin{bmatrix} \dfrac{1}{s_{hc}} & 0 & 0 \\ 0 & \dfrac{1}{s_{hc}} & 0 \\ 0 & 0 & \dfrac{1}{s_{vc}} \end{bmatrix}$$

$$p = Wr$$

$$\|Up\| = \|UWr\| = 1$$

$$\|Up\|^2 = \|UWr\|^2 = 1$$

Therefore : $r^T U^T W^T W U r = 1$

with :

if for example $W^T W = \begin{bmatrix} a & b & d \\ b & c & e \\ d & e & f \end{bmatrix}$

$$a\frac{x^2}{s_{hc}^2} + 2b\frac{xy}{s_{hc}^2} + 2d\frac{xz}{s_{hc}s_{vc}} + 2e\frac{yz}{s_{hc}s_{vc}} + c\frac{y^2}{s_{hc}^2} + f\frac{z^2}{s_{vc}^2} = 1$$

The minimum and maximum value of $\Delta x_c$, $\Delta y_c$, $\Delta z_c$ can be determined by setting the three discriminants of the above quadratic equation equal to zero, and then considering them as three new quadratic equations.

So :

$$\Delta x_c = \pm\sqrt{\frac{\left(e^2 - fc\right)s_{hc}^2}{a\left(e^2 - fc\right) + cd^2 + fb^2 - 2bde}}$$

$$\Delta y_c = \pm\sqrt{\frac{\left(d^2 - af\right)s_{hc}^2}{c\left(d^2 - af\right) + ae^2 + cd^2 - 2bde}}$$

$$\Delta z_c = \pm\sqrt{\frac{\left(b^2 - ac\right)s_{vc}^2}{ae^2 - 2bde + cd^2 + f\left(b^2 - ac\right)}}$$

The horizontal conflict probability can be written as follows :

$$P_{hc} = \int_{-\Delta y - \Delta y_c}^{-\Delta y + \Delta y_c} \int_{-\infty}^{+\infty} p(x,y)dxdy = \int_{-\Delta y - \Delta y_c}^{-\Delta y + \Delta y_c} p(y)dy \int_{-\infty}^{+\infty} p(x)dx = \int_{-\Delta y - \Delta y_c}^{-\Delta y + \Delta y_c} p(y)dy$$

$$P_{hc} = P(-\Delta y + \Delta y_c) - P(-\Delta y - \Delta y_c)$$

The vertical conflict probability is equal to :

$$P_{vc} = \begin{cases} 0 \text{ if } \left\{\left|z_1 - z_2\right| - \left(s_{vp1} + s_{vp2}\right) > s_{vc}\right\} \\ 1 \text{ if } \left\{\left|z_1 - z_2\right| - \left(s_{vp1} + s_{vp2}\right) \le s_{vc}\right\} \end{cases}$$

The total conflict probability is : $P_c = P_{hc} * P_{vc}$

For non level fight, an exact analytical solution has not been found. But, a very good approximation can be determined using a bounding rectangle around the cylindrical conflict zone as illustrated in figure 11.

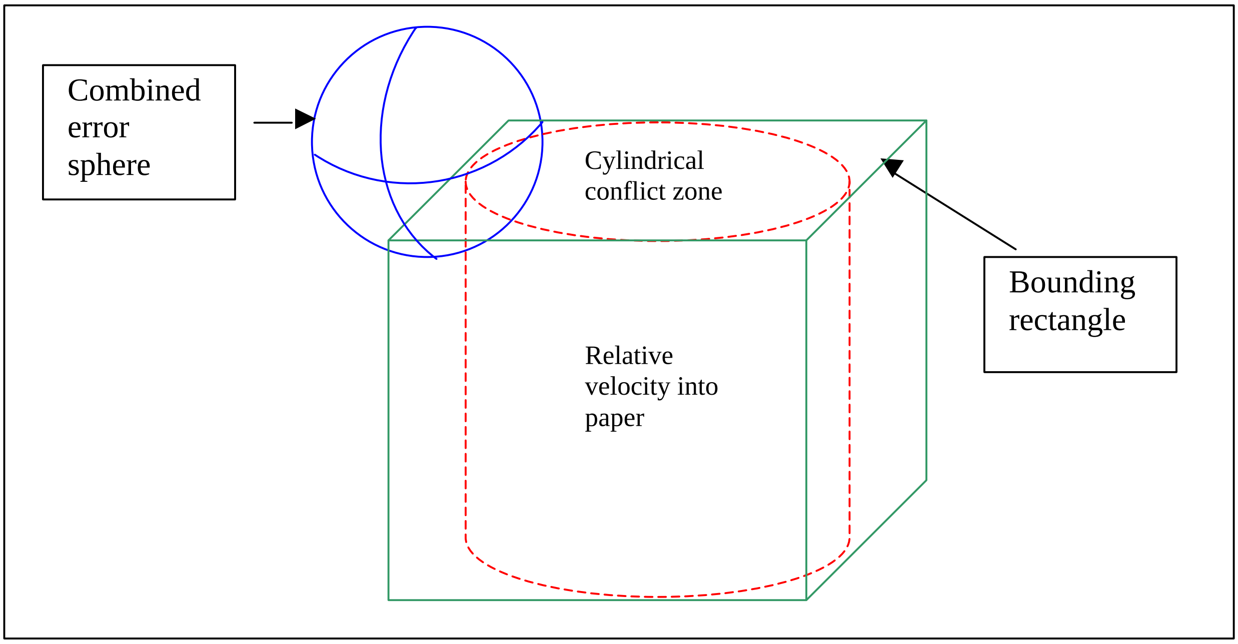


**Figure 11 : Transformed 3D encounter geometry**

The expression of the conflict probability $P_c$ can be simplified as follows :

$$P_c = \int_{-\Delta z-\Delta z_c}^{-\Delta z+\Delta z_c} \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} \int_{-\infty}^{+\infty} p(x,y,z)dxdydz$$

$$= \int_{-\Delta z-\Delta z_c}^{-\Delta z+\Delta z_c} p(z)dz \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} p(y)dy \int_{-\infty}^{+\infty} p(x)dx = \int_{-\Delta z-\Delta z_c}^{-\Delta z+\Delta z_c} p(z)dz \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} p(y)dy$$

$$P_c = \left(P(-\Delta z+\Delta z_c) - P(-\Delta z-\Delta z_c)\right)\left(P(-\Delta y+\Delta y_c) - P(-\Delta y-\Delta y_c)\right)$$

$$P_c = P_{vc} * P_{hc}$$

with $\Delta y_c$ , $\Delta z_c$ given previously and :

$$\Delta y = y_1 - y_2$$

$$\Delta z = z_1 - z_2$$

# 6. More realistic modelling of flights in 3D: turns are instantaneous at fixed times

The aircraft fly with winds, change of velocities, headings and altitudes.

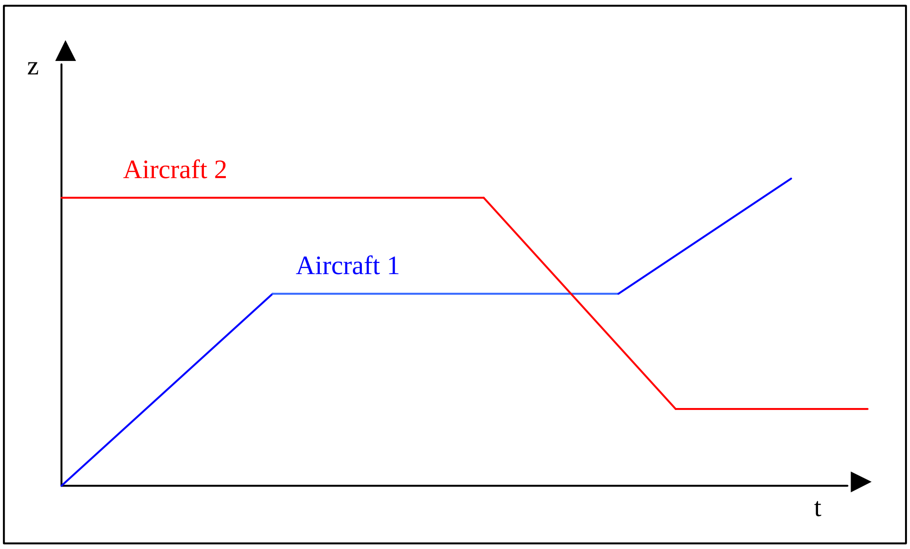


**Figure 12: A general case in 3D**

As before, for each aircraft, new trajectory points are calculated to have the same time step. That is to say, the predicted states of an aircraft when the other flies over a TCP are known.

## 6.1. Assumptions and Combined Covariance

The notations and the methods are identical to those used in part 5 but here, the wind is taken into consideration.

Let $\sigma_{hwj}$ be the horizontal wind resulting error and $\sigma_{vwj}$ be the vertical one.

As in 2D, it is assumed that the angle between the vector airspeed and the aircraft centre line (angle of attack) is always zero. Therefore, the trajectory prediction error for an aircraft will be modelled as normally distributed, with zero mean and with a covariance matrix that has eigenvectors in the along-track, cross-track and vertical directions.

It is assumed for simplicity that the changes of velocity are instantaneous.

The time $t_i$ is variable, but, for simplicity, it is assumed to be fixed at its mean value. Now verifying the combined error co-variance matrix calculation :

$$t \in [t_i ; t_{i+1}[$$

$$Q^j(t) = R_i^j\left(s_{hpj}^2, s_{hpj}^2, s_{vpj}^2\right) I R_i^{jT} + \sum_{k=0}^{i-1} \left(t_{k+1} - t_k\right) A_k^j + \left(t - t_i\right) A_i^j$$

where :

$$A_k^j = R_k^j \left( \left(\sigma_{hvj}^{k}\right)^2 + \left(\sigma_{hwj}^{k}\right)^2, 0, \left(\sigma_{vvj}^{k}\right)^2 + \left(\sigma_{vwj}^{k}\right)^2 \right) I R_k^{jT}$$

and :

$$R_k^j = \begin{bmatrix} \cos\psi_k^j \cos\theta_k^j & -\sin\psi_k^j & \cos\psi_k^j \sin\theta_k^j \\ \sin\psi_k^j \sin\theta_k^j & \cos\psi_k^j & \sin\psi_k^j \sin\theta_k^j \\ -\sin\theta_k^j & 0 & \cos\theta_k^j \end{bmatrix}$$

Therefore the combined covariance is matrix : $M = Q^1 + Q^2 - Q^{12}$

As seen in Section 6, $Q^{12}$ is the cross corelation term. If the two aircraft are far from each other, it is negligible. Otherwise, it must be calculated. The mean direction of wind, can be used to check if the two aircraft are deviating in the same direction or in opposite directions.

LEMMA 6 :

> *$Q^{12}(t)$=*
>
> $$\pm\left(\sigma_{hw1}\sigma_{hw2}\left(\vec{e}_1\vec{e}_2^T + \vec{e}_2\vec{e}_1^T\right) + \sigma_{hw1}\sigma_{vw2}\left(\vec{e}_1\vec{\zeta}^T + \vec{\zeta}\vec{e}_1^T\right) + \sigma_{vw1}\sigma_{hw2}\left(\vec{e}_2\vec{\zeta}^T + \vec{\zeta}\vec{e}_2^T\right) + \sigma_{vw1}\sigma_{vw2}\left(2\vec{\zeta}\vec{\zeta}^T\right)\right)t$$
>
> with $\vec{e}_1$ be the normalised direction vector of aircraft 1 and $\vec{e}_2$ be the normalised direction vector of aircraft 2.

*Proof : Appendix A*

Then , the total encounter correlation matrix is :

$$\begin{aligned} Q^{12} = \sum_{k=0}^{i-1} \{ \pm \{ &\sigma_{hw1}^{k}\sigma_{hw2}^{k}\left(\vec{e}_1^{\,k}\vec{e}_2^{\,kT} + \vec{e}_2^{\,k}\vec{e}_1^{\,kT}\right) + \sigma_{hw1}^{k}\sigma_{vw2}^{k}\left(\vec{e}_1^{\,k}\vec{\zeta}^T + \vec{\zeta}\vec{e}_1^{\,kT}\right) + \sigma_{vw1}^{k}\sigma_{hw2}^{k}\left(\vec{e}_2^{\,k}\vec{\zeta}^T + \vec{\zeta}\vec{e}_2^{\,kT}\right) \\ &+ \sigma_{vw1}^{k}\sigma_{vw2}^{k}\left(2\vec{\zeta}\vec{\zeta}^T\right)\}\left(t_{k+1} - t_k\right)\} \pm \{ \sigma_{hw1}^{i}\sigma_{hw2}^{i}\left(\vec{e}_1^{\,i}\vec{e}_2^{\,iT} + \vec{e}_2^{\,i}\vec{e}_1^{\,iT}\right) + \sigma_{hw1}^{i}\sigma_{vw2}^{i}\left(\vec{e}_1^{\,i}\vec{\zeta}^T + \vec{\zeta}\vec{e}_1^{\,iT}\right) \\ &+ \sigma_{vw1}^{i}\sigma_{hw2}^{i}\left(\vec{e}_2^{\,i}\vec{\zeta}^T + \vec{\zeta}\vec{e}_2^{\,iT}\right) + \sigma_{vw1}^{i}\sigma_{vw2}^{i}\left(2\vec{\zeta}\vec{\zeta}^T\right)\}\left(t - t_i\right) \end{aligned}$$

## 6.2. CONFLICT PROBABILITY AT FIXED TIMES

For each time step, three different possible kinds of encounter can occur (figure 13) :

-first, a 2D encounter (curves 2&5, 4&6)

-secondly, a 3D encounter viewed as a 2D encounter (curves 1&2, 1&5, 1&4, 1&6, 2&4, 2&6, 5&4, 5&6)

-thirdly, a real 3D encounter (curves 1&3)

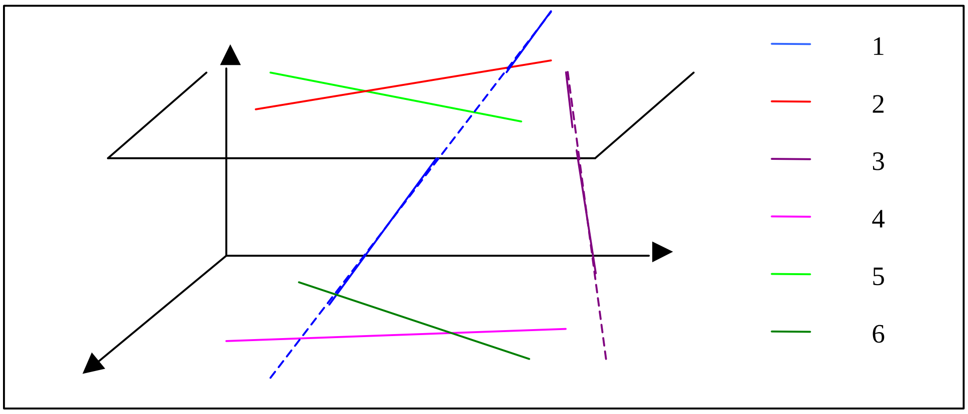


**Figure 13: different possible encounters**

### 6.2.1. A 2D ENCOUNTER

The conflict probability on this segment is :

$$P_c(t \in [t_i;t_{i+1}[) = \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} \int_{-\Delta x-\Delta x'}^{-\Delta x+\Delta x''} p(x,y)dxdy = \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} p(y)dy \int_{-\Delta x-\Delta x'}^{-\Delta x+\Delta x''} p(x)dx$$

$$P_c(t \in [t_i;t_{i+1}[) = P(-\Delta y + \Delta y_c)P(-\Delta x + \Delta x'') - P(-\Delta y - \Delta y_c)P(-\Delta x - \Delta x')$$

where :

- $\Delta x = x_1 - x_2$ , $\Delta y = y_1 - y_2$ are respectively the x and *y*-co-ordinates of the stochastic aircraft with respect to the reference aircraft.
- P is the cumulative normal probability function.
- $\Delta x' = (s_c\sqrt{c/ac-b^2}) + (t_m - t_i)\Delta v_i$
- $\Delta x'' = \left(s_c\sqrt{c/ac-b^2}\right) + \left(t_{i+1} - t_m\right)\Delta v_i$

### 6.2.2. A 3D ENCOUNTER VIEWED AS A 2D ENCOUNTER

$$\Delta x_c = \pm\sqrt{\frac{\left(e^2 - fc\right)s_{hc}^2}{a\left(e^2 - fc\right) + cd^2 + fb^2 - 2bde}}$$

$$\Delta y_c = \pm\sqrt{\frac{\left(d^2 - af\right)s_{hc}^2}{c\left(d^2 - af\right) + ae^2 + cd^2 - 2bde}}$$

$$\Delta z_c = \pm\sqrt{\frac{\left(b^2 - ac\right)s_{vc}^2}{ae^2 - 2bde + cd^2 + f\left(b^2 - ac\right)}}$$

$$\Delta x' = \Delta x_c + (t_m - t_i)v_i$$
$$\Delta x'' = \Delta x_c + (t_{i+1} - t_m)$$

The horizontal conflict probability on this segment can be written as follows:

$$P_{hc}(t \in [t_i;t_{i+1}[) = \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} \int_{-\Delta x-\Delta x'}^{-\Delta x+\Delta x''} p(x,y)dxdy = \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} p(y)dy \int_{-\Delta x-\Delta x'}^{-\Delta x+\Delta x''} p(x)dx$$
$$P_{hc}(t \in [t_i;t_{i+1}[) = (P(-\Delta y + \Delta y_c) - P(-\Delta y - \Delta y_c))(P(-\Delta x + \Delta x'') - P(-\Delta x - \Delta x'))$$

The vertical conflict probability on this segment is equal to :

$$P_{vc}(t \in [t_i;t_{i+1}[) = \begin{cases} 0 \text{ if } \{|z_1 - z_2| - (s_{vp1} + s_{vp2}) > s_{vc}\} \\ 1 \text{ if } \{|z_1 - z_2| - (s_{vp1} + s_{vp2}) \le s_{vc}\} \end{cases}$$

The total conflict probability is : $P_c(t \in [t_i;t_{i+1}[) = P_{hc}(t \in [t_i;t_{i+1}[) * P_{vc}(t \in [t_i;t_{i+1}[)$

### 6.2.3. A REAL 3D ENCOUNTER

The expression of the conflict probability $P_c$ on this segment can be simplified as follows :

$$P_c(t \in [t_i;t_{i+1}[) = \int_{-\Delta z-\Delta z_c}^{-\Delta z+\Delta z_c} \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} \int_{-\Delta x-\Delta x'}^{-\Delta x+\Delta x''} p(x,y,z)dxdydz$$
$$= \int_{-\Delta z-\Delta z_c}^{-\Delta z+\Delta z_c} p(z)dz \int_{-\Delta y-\Delta y_c}^{-\Delta y+\Delta y_c} p(y)dy \int_{-\Delta x-\Delta x'}^{-\Delta x+\Delta x''} p(x)dx$$
$$P_c(t \in [t_i;t_{i+1}[) = (P(-\Delta z + \Delta z_c) - P(-\Delta z - \Delta z_c)) * (P(-\Delta y + \Delta y_c) - P(-\Delta y - \Delta y_c))(P(-\Delta x + \Delta x'') - P(-\Delta x - \Delta x'))$$
$$P_c(t \in [t_i;t_{i+1}[) = P_{vc}(t \in [t_i;t_{i+1}[) * P_{hc}(t \in [t_i;t_{i+1}[)$$

## 6.3. THE TOTAL CONFLICT PROBABILITY

Then the total conflict probability can be bounded as follows :

$$\max_i P_c(t \in [t_i;t_{i+1}[) < P_c \le \sum_i P_c(t \in [t_i;t_{i+1}[)$$

But, it is more interesting to look at the worst case conflict probability on a segment :

$$P_{worst} = \max_i P_c(t \in [t_i;t_{i+1}[)$$

# 7. EXPERIMENTAL RESULTS

H. Erzberger and R. Paielli 's modelling of position error results from a statistical study. The model predictions were validated with a variety of different live data. So these results are a good illustration of reality and also a good reference for this work. Therefore some of their numerical examples will be reproduced.

Then the numerical examples are repeated with the suggested enhanced modelling of position error. It could be that the error made is constant on average but with 'large variations'. The error of position resulting from velocity is chosen to be modelled as a Brownian process. This error can be explained for example by an inaccuracy in instruments. This leads to some important differences in the 'precision' of the two detection methods but not in the principles.

A set of numerical examples of conflict probabilities and related quantities are generated as a function encounter geometry. The aircraft speeds are 480kts (≈ 247m/s) in every case. The conflict separation distance is 5nmi. The cross-track rms. error is 1nmi (1852m), and the along-track rms. error started at zero to grow linearly at a rate of 15kts (≈ 7.7m/s). Wind-error cross correlation between aircraft is not modelled.

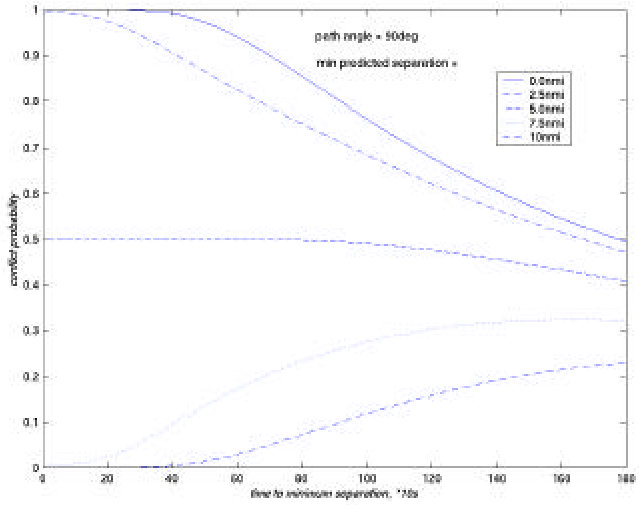


**Figure 14 : Effect of mimimun predicted separation with H. Erzberger and R. Paielli model**

Figure 14 shows the effects of minimum predicted separation as a function of time on CPA, with the distance at CPA as a parameter, where the path-crossing angle is 90deg.

For small prediction times, the co-variances are small and the conflict probabilities are a strong function of distance at CPA. That is to say, near the CPA, the conflict probability depends almost totally on the predicted minimum separation. For larger prediction times, the co-variances grow and the conflict probabilities become a weaker function of distance at CPA. The conflict probabilities converge and asymptotically approach zero as prediction time increases.

The error could be modelled as a Brownian process. The mean of the Brownian here is taken as zero. For the same data as in the previous example, (same along-track rms. and error rate), figure 15 shows the effect of minimum predicted separation for the values 0nmi to 10nmi.

The combined matrix co-variance grows with time less quickly than in the previous model. That is why the conflict probabilities are a strong function of distance at CPA for large prediction times.

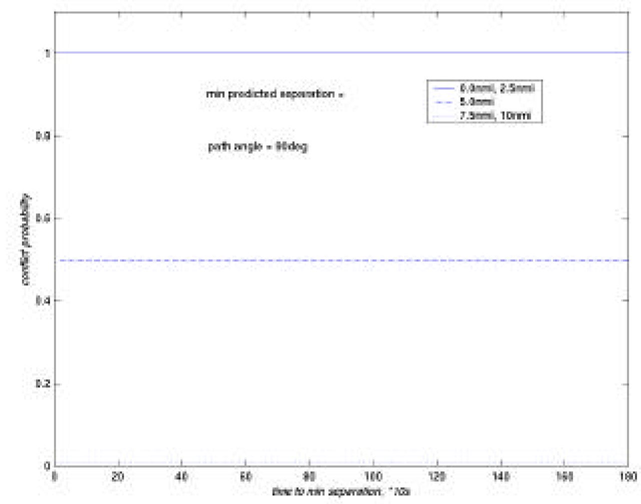


**Figure 15 : Effect of minimun predicted separation with the velocity error as a Brownian**

A comparison between figure 14 and figure 15 show that the enhanced modelling of velocity gives the ability to detect conflicts for the next 30 minutes ahead instead of 5 or 10 minutes. This is not realistic. Indeed, there is a non-zero probability that at least one of the aircraft will change its flight plan during these 30 min. So the detection of conflict can not be sure for a long period. Furthermore, the rate of along track rms. error value was chosen equal to the one used in the Erzberger and Paielli model, and that can not be.

Indeed, the different models of error can not be directly compared because they are not using the same scales. But their consequences on the conflict detection can be studied and compared. To test the new position error model, the experimental results seen in figure 14 are reproduced with the same data, except those concerning the error on the along-track position. It will be done by tuning of the to duplicate the results. So some values of the position error rms. were tried. When the conflict probability at CPA and 30 min to CPA were equal to those found with the reference results and the behaviour of the curves were identical, the error rate was chosen to be presented here in figure 16.

The rms. error on velocity is here equal to 325m/s or 632kts. So it is upper than the true speed of the aircraft (247m/s or 480kts). This is not a proportion physically realistic. More this model detect worse the conflict than the reference model. That is to say it detects the conflict later. So it is not the dynamic model corresponding to the reference model. But it can be used to partially characterise the error of position.

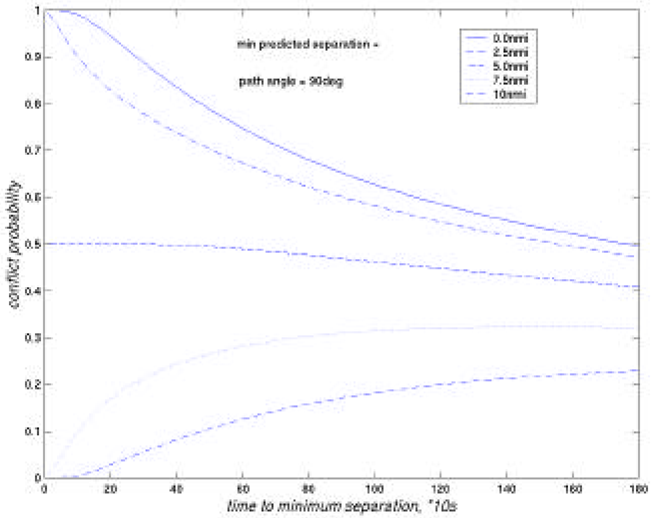


**Figure 16 : Effects of min predicted separation with the velocity error modelled as a Brownian**

Figure 17 shows the effect of prediction error growth rate on conflict probability. The error rate is taken to be 50% of the nominal velocity (here equal to 240kts or 123.5m/s). For this value, the conflict probability is plotted again as a function of the time to CPA for minimum predicted separation of 0nmi, 2.5nmi, 5nmi, 7.5nmi and 10nmi.

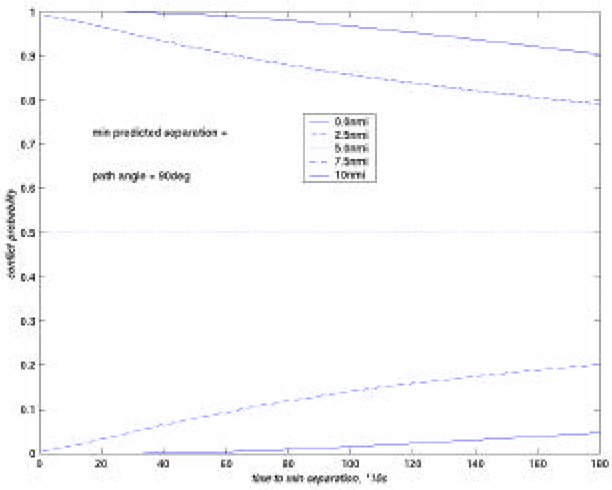


**Figure 17 : Effect of prediction error growth rate with the velocity error as a Brownian**

It also illustrates some important characteristics. For a distance at CPA substantially less than minimum allowed separation, the conflict probability starts at unity and decreases monotonically as a function of prediction time. The effect of larger error growth rates is to cause the conflict probability to decrease more rapidly. Symmetrically for a distance at CPA substantially greater than the minimum allowed separation, the conflict probability starts at zero and increases to some maximum value, to finally decreases back toward zero.

This occurs because the individual error ellipses expand and start to overlap, but then after the maximum conflict probability they expand even more and the probability density function becomes flatter. The effect of lager growth rates is to cause the conflict probability to increase more rapidly and then after the maximum, to decrease more rapidly.

Figure 18 shows the sensitivity of the prediction error growth rate's effects, the conflict probability is plotted again as a function of the time to CPA, but for a minimum predicted separation of 2.5nmi, 5nmi and 7.5nmi. Then the along track rms. error growth rate is varied from 0m/s to 247m/s (480kts) by steps of 61.75m/s (120kts). So the error rate varies from 0 to 100% of the velocity nominal value by steps of 25%.

It can be seen that this modelling is not really sensitive to an error rate smaller than 25% of the velocity's nominal value. For the special case where the minimum predicted separation is equal to the separation minima, the conflict probability is not sensitive to the error rate variations.

To reproduce Erzberger and Paielli's results, it is necessary to find an error rate value with the following properties. The conflict probabilities at CPA and at 30mn to CPA must be equal to those found with the reference model. And the behaviour of the curves must be identical.

It can be seen that even for an error rate equal to 100% of the nominal velocity, the detection of conflict is better at 30mn to CPA than in the reference model. And then close to CPA, it becomes worse.

So this modelling is not a good improvement over the reference error modelling.

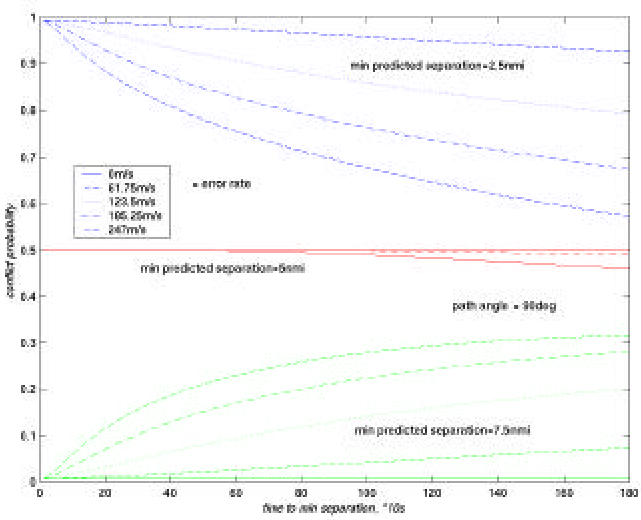


**Figure 18 : Sensitivity to the prediction error growth rate with the velocity error as a Brownian**

Figure 19 shows the effect of path crossing angle on conflict probability when H. Erzberger and R. Paielli's method is used. Conflict probability is plotted as a function of time to CPA, but with the path-crossing angle as a parameter where the distance at CPA is zero nautical miles. As the point of reference, the curve for the path crossing angle of 90deg is a repeat of the corresponding curve in figure 14. As the prediction time increases, the conflict probability decreases faster for smaller path-crossing angles. So the fastest decrease of the curve is for a path-crossing angle of 45deg.

For small path-crossing angles, the aircraft take more time to fly over each other, that is why the conflict probabilities tend to stay at one for longer.

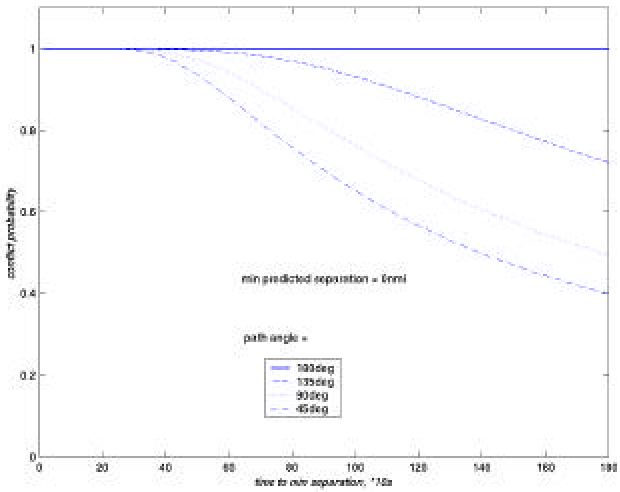


**Figure 19 : Effect of path crossing angle with H. Erzberger and R. Paielli model**

If cross-correlation of winds exist, they are taken into account, however these curves could be significantly different for path-crossing angles less than about 45deg. In this case, a portion of the trajectory-prediction error would cancel with the position difference, and the effective error growth rate would be smaller. Hence the conflict probabilities for such small path angles would not decrease as rapidly as shown in figure 19.

As seen before, when the model of velocity error is used, conflict detection is a strong function of the distance at CPA. Because in this example the distance at CPA is equal to zero, for all path-crossing angles, the probabilities of conflict are equal to one all the time.

Figure 20 shows the effect of constant cruise speed differences for small (15deg) encounter angles. All the curves are based on a predicted collision at constant altitude. One aircraft flies at 500kts (257m/s) while the speed of the other aircraft is reduced in steps of 50kts (26m/s) down to 300kts (154m/s). The figure shows that conflict probabilities decrease less rapidly with prediction time for larger speed differences. This behaviour can be understood by visualising the encounter from the perspective of an observer riding on one of the aircraft. Having a large difference in velocity at a shallow angle is similar to the case in which a faster aircraft overtakes a slower one on a converging path, diminishing the effect of random along-track errors.

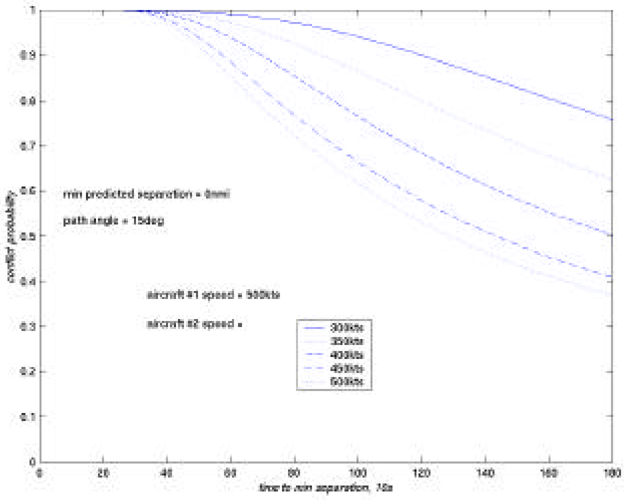


**Figure 20: Effect of speed difference with H. Erzberger and R. Paielli model**

As before, with this model, conflict detection is a strong function of the distance at CPA. Because in this example the distance at CPA is equal to zero, for all velocities, the probabilities of conflict are equal to one all the time.

The results presented in this section show some important differences between the two detection methods. These differences a result of the error modelling used.

It had been seen that the use of a Brownian to model the position error is not adapted. But it can be used in complement of an other position modelling. Therefore a new position error model that could be closer to reality is needed to improve the conflict detection.

# 8. ENHANCEMENT OF THE POSITION ERROR MODELLING

A good error modelling is a major point in probabilistic studies. Indeed that makes all the difference between a valid prediction and a random prediction. More, it ensures the precision of the conflict detection. It had been seen that according to H. Erzberger and R. Paielli the position error could be modelled as normally distributed with a constant rate that linearly grows with time. Among that, a question appears : how can the dynamic model of the position error be made explicit?

A focus on the error model will be done in this part. New models will be proposed and discussed. The results of H. Erzberger and R. Paielli presented in the previous part will be considered, in a certain way, as a reference for this study. Their results were validated on live data and are also a good representation of the reality.

As seen in the previous section, the different models of error can not be directly compared because they are not using the same kind of scales. But their consequences on the conflict detection can be studied and compared. To test the new position error models, the experimental results seen in the figure 14 are reproduced with the same data, except those concerning the error on the along-track position. Tuning of the model to duplicate the results will do it. So some values of the position error rms. were tried. When the conflict probability at CPA and 30min to CPA were equal to those found with the reference results and the behaviour of the curves were identical, the error rates were chosen to be presented here.

## 8.1. ACCELERATION ERROR MODELLED AS A BROWNIAN PROCESS

The position error can be the result of an error in the acceleration possibly caused by wind.

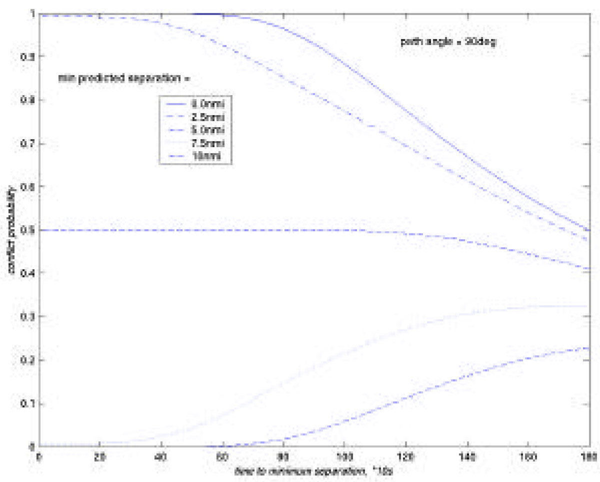


**Figure 21 : Effect of minimum predicted separation with the acceleration error as a Brownian**

By modelling the acceleration as a Brownian process the experimental results of Erzberger and Paielli can be reproduced.

For the same data as in figure 15, except the along-track error rate rms. (which is here $0.31 m/s^2$) the calculation of the effect of minimum predicted separation with this modelling gives almost the same conflict detection (figure 21).

Nevertheless the detection of conflicts is better. That is to say, they are detected on average 5mn earlier than with Erzberger and Paielli's modelling. This difference could be explained as follows. The aircraft navigation accuracy is taken into account more precisely, so the error in position can be better estimated.

## 8.2. ACCELERATION ERROR COUPLED WITH VELOCITY ERROR

As seen before, the acceleration error is modelled as a Brownian process. Then a velocity error is added. There are two cases : the position error can be with a constant rate or resulting of a velocity error modelled as a Brownian process.

These different cases can be explained as follows : the error of position can be the result of uncompensated wind effect errors and/or instruments errors.

The error rate values are chosen to reproduce the results of H. Erzberger and R. Paielli. That is to say, some different values were tried. As before only the values that give the closest results, are presented here.

First, the results with the velocity error modelled as a Brownian process are presented.

Figure 22 shows the effects of minimum predicted separation for the values 0nmi to 10nmi in steps of 2.5nmi. The acceleration error rate is $0.31 m/s^2$ and the velocity error rate is 7.52m/s.

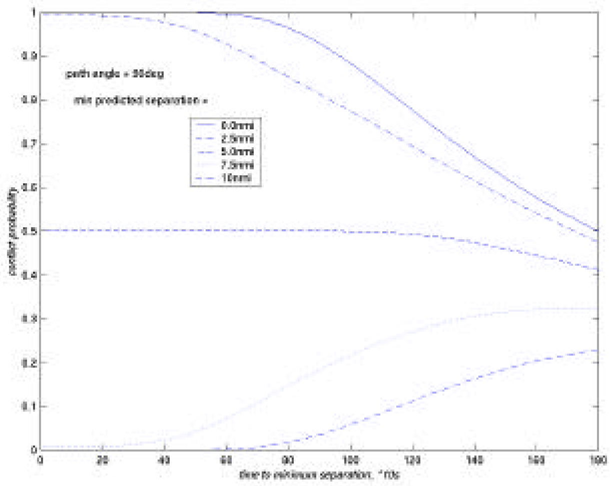


**Figure 22 : Effect of minimum predicted separation with the acceleration error and the velocity error as a Brownian**

When a comparison is made with the model seen in figure 21, the results are identical. The error in velocity does not degrade the conflict detection much when it is not too large. So by adding it to the acceleration error, it does not modify the results greatly.

Nevertheless it could be interesting to determine when it starts to have an influence on the conflict detection.

Figure 23 shows the effect of prediction error growth rate for a minimum predicted separation of 2.5nmi, 5nmi and 7.5nmi. Conflict probability is plotted again as a function of the time to CPA but with the velocity rms. error as parameter. Here it varies from 0kts to 480kts with a step of 240kts. That is to say, the error on velocity varies from 0 to 100% of the nominal velocity value by steps of 50%. The variations of the curve are not so large in comparison with the greatness of the errors made.

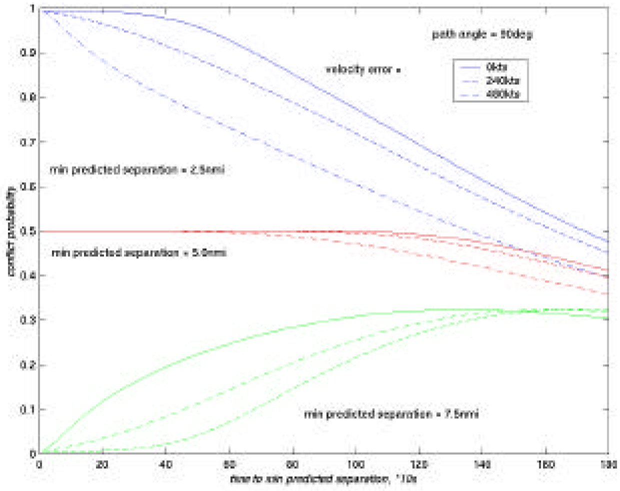


**Figure 23 : Effect of prediction error growth rate with the acceleration error and the velocity error as a Brownian**

Secondly, the results with the reference position error model are presented.

Figure 24 shows the effects of minimum predicted separation for the values 0nmi, 2.5nmi, 7.5nmi, 10nmi. The acceleration error rate is $0.17m/s^2$ and the velocity error rate is 6.2m/s. When a comparison is made with the previous models, the results are identical to Erzberger and Paielli's.

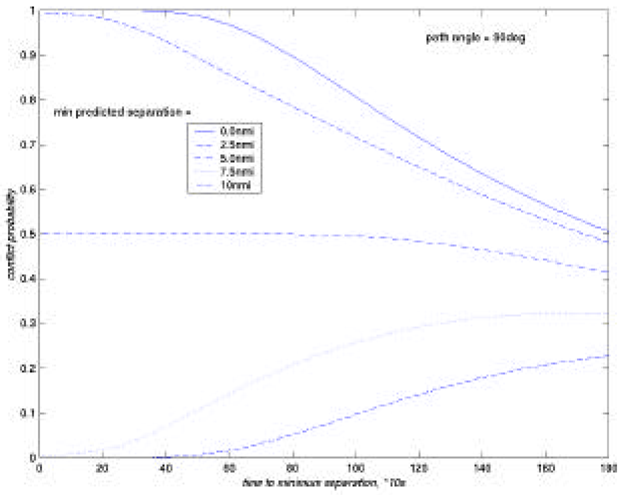


**Figure 24 : Effect of minimum predicted separation with the acceleration error as a Brownian and the velocity error as a constant**

Figure 25 shows the effect of prediction error growth rate with a minimum predicted separation of 0nmi. The velocity error rate varies from 0 to 15m/s with a step of 5m/s. That is to say that the error on velocity varies from 0 to 6% of the nominal velocity value. The variations of the curve are large in comparison with the small size of the errors made. This can be explained by the fact that with a constant error and with large prediction times the conflict detection is a weak function of the distance at CPA and a strong function of the time to CPA.

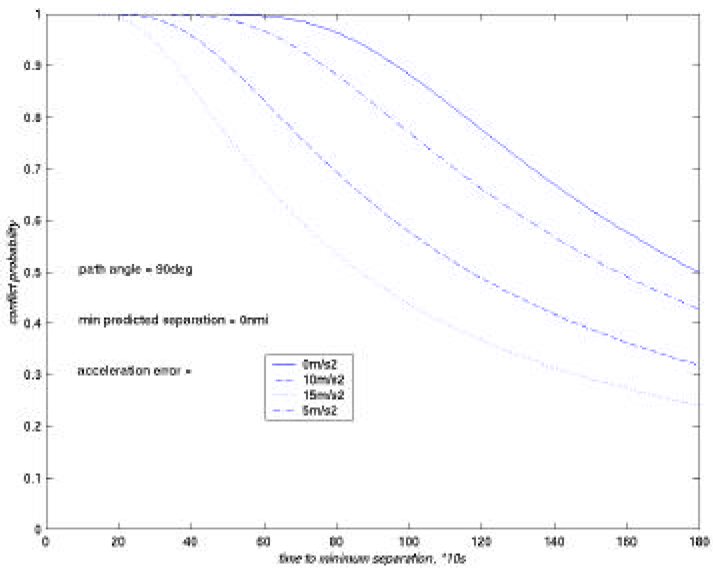


**Figure 25 : Effect of prediction error growth rate with the acceleration error as a Brownian and the velocity error as a constant**

## 8.3. ACCELERATION ERROR COUPLED WITH A CONSTANT VELOCITY ERROR AND A VELOCITY ERROR MODELLED AS A BROWNIAN PROCESS

The error in position can be constructed by the three different modelling methods seen so far. Actually, this gives results closed to the Erzberger and Paielli's ones.

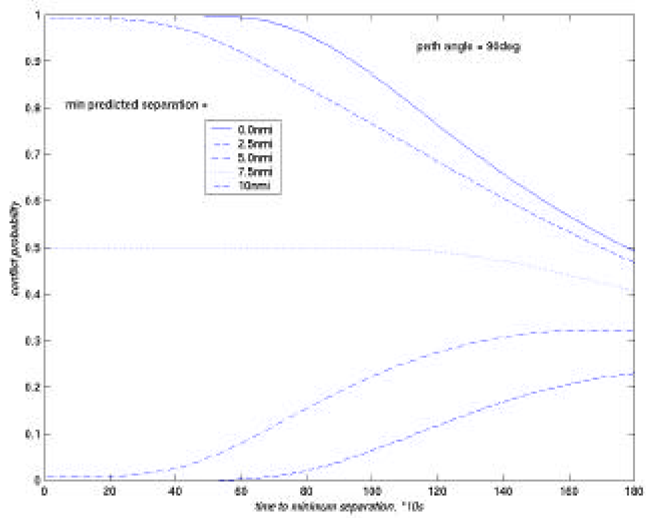


**Figure 26 : Effect of minimum predicted separation with the acceleration error as a Brownian, a part of velocity error as constant and an other part of velocity error as Brownian**

Figure 26 shows the effects of minimum predicted separation for the values 0 and 2.5nmi. The acceleration error rate is 0.18m/s$^2$, the constant velocity error rate is 1.5m/s and the variance rms. of the velocity as a Brownian is 5m/s.

Note that the conflicts are better detected than with Erzberger and Paielli's modelling. Indeed they are detected in average 5 minutes earlier. As seen before, this difference could be the result of the more precise prediction of position error.

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# 9. CONCLUSION

The research work described in this report is based upon H. Erzberger and R. Paielli's papers [4,5]. Several improvements have been considered :

- a more realistic modelling of error
- a study of wind-error cross-correlation model
- a modelling of turns and velocity changes in 2D and 3D

Some different position error models have been proposed to provide a reliable conflict estimation. Beyond, an appropriate modelling of uncertainty should be a compromise between three considerations : (1) a position error normally distributed with a constant rate that grows linearly with time, (2) a position error resulting from a velocity error modelled as a Brownian process, and (3) a position error resulting from an acceleration error modelled as a Brownian process. A mixed of the three models presented in this paper is a possible solution. To weight each model, the aircraft and its own characteristics can be considered. This will assure more flexibility and suitability in term of modelling. In this way it will improve the conflict detection accuracy.

Nevertheless, it is necessary to experiment with different possibilities to discover the better-mixed model. Some experiments and simulations are needed to validate these models. If a good model is found, some technological questions appear. For example : can the actual technology provide precise rms. error rates to justify the use of this exact modelling?

Therefore, three key points that can profit directly by this new error model have been identified. The first one is to identify, integrate and model all the uncertainties that affect 'significantly' trajectory predictions. The second point is to provide capabilities not only for conflict detection but also for monitoring potential or solved conflicts, typically through the estimation of minimum distance. The third point identified is to allow the modelling of uncertainty under different assumptions and for different applications and scenarios. For instance, two typical scenarios are of interest: on one side aircraft flying with auto-pilot (A/P) and transmitting flight states, and on the other, aircraft flying with Flight Management System (FMS) and transmitting full intent (or Trajectory Change Points, TCPs).

With the developed method, the probability of conflict for aircraft pairs has been obtained greater efficiency and accuracy.

These studies lead to a better prediction of conflict and further developments could even improve the quality of detection by taking into account more realistic trajectories, velocities, and changes in automatic or manual navigation modes. To validate them the implementation of these different models be will necessary.

False alert rate research can be also a guide to the worth of a conflict detection method. The model limits can be tested by studying a couple of aircraft and by making all flight parameters vary one by one. Then, by increasing the number of aircraft the consequences of the traffic density on the precision of the conflict detection can be analysed.

This will lead to define new types that will help to classify priority level for the resolution of conflicts.

As the result of this classification, procedures could be devised to resolve conflicts. First, the trajectories of all aircraft in the region of interest are predicted for approximately the next 10 to 20 minutes. Those deterministic predictions are based on current estimated positions, velocities, flights plans, and predicted winds aloft. The second step is to coarsely screen all possible aircraft pairs to eliminate those with a negligible possibility of conflict (for example when the aircraft are in different areas). The third step is to estimate the conflict probability for those remaining aircraft pairs. This probability is based both on predicted trajectories and on an estimate of their uncertainty. Then, the conflict probability should be studied and then the priority level of conflict resolution calculated.

When the model is defined and the conflict detected, a corresponding problem is sought For that criteria that translate the needs and the expectations of the user are required The relative importance of each criterion (i.e. safety, passenger comfort, fuel or aircraft cost, manoeuvring complexity…) will be adjusted to lead to an optimisation problem. Then to solve this optimisation problem, a method has to be chosen.

Typically we can look for a possible solution in two different ways. One way is to use stochastic algorithms. The other way is to find a realistic relaxation of constraints and to solve it with stochastic algorithms or branch and bound.  Then, one single solution (with the possibility to provide the pilot all the solutions or only one of them) can be chosen. This solution would consist of minimum cost manoeuvres and optimal times to initiate, end and return to a nominal trajectory. This would be a trade-off between efficiency and certainty.

For multi-aircraft conflicts and decentralised decisions, protocols of co-ordination must be defined.

The simplest idea is to assign a resolution order with the help of a counter. Thus, when an aircraft has the counter it can chose the manoeuvre it wants with respect to the manoeuvres previously chosen by the aircraft having priority.

It seems also that game theory can help in the tasks repartition and the order of execution. Actually, the problem can be modelled as a n players game (n= number of planes in a considered air-space) [1]. First, the game is non-co-operative, each player (aircraft) considers the others as a disturbance and tries to adopt an optimal resolution strategy for the worst possible action of the others. If there is still no safe solution, then coalitions can be introduced. Finally, if it is still unsafe, a co-operative solution is necessary.

Further developments of the study and refining this model will be required for a better management of flight conflicts and to cope with the increasing air traffic of the next decade.

# APPENDIX A : PROOFS

**Proof lemma 1 :**

Let X(t) and Y(t) be the trajectories of the aircraft beginning at time $t_0$ .

Let $V^1$ and $V^2$ be their velocities.

Let $\Delta p_0$ be the position difference at time $t_0$, and $\Delta v$ the constant velocity difference.

$$\vec{X}(t) = \vec{X}_0 + t\vec{V}^1$$

$$\vec{Y}(t) = \vec{Y}_0 + t\vec{V}^2$$

The distance d between the two paths is : $\left\|\vec{X}(t) - \vec{Y}(t)\right\|$

we want to find :

$$\min_t \left\|\vec{X}(t) - \vec{Y}(t)\right\|$$

But for simplicity we can check for the square distance minimum, it will give us the same result.

$$d^2 = \left(\vec{X}(t) - \vec{Y}(t)\right)^T . \left(\vec{X}(t) - \vec{Y}(t)\right)$$

$$d^2 = \left(\Delta\vec{p}_0 + t\Delta\vec{v}\right)^T . \left(\Delta\vec{p}_0 + t\Delta\vec{v}\right)$$

$$d^2 = \Delta\vec{p}_0^{\,T}\Delta\vec{p}_0 + t\Delta\vec{p}_0^{\,T}\Delta\vec{v} + t\Delta\vec{v}^T\Delta\vec{p}_0 + t^2\Delta\vec{v}^T\Delta\vec{v}$$

by derivation, we obtain :

for $V^1$ equal to $V^2$, d is constant so there is no tCPA.

for $V^1$ different from $V^2$, $t = -\dfrac{\Delta p_0^T \Delta v}{\Delta v^T \Delta v}$

$$tCPA = t_0 + t \quad \square$$

**Justification 1 :**

$$P\left(\exists t, d\left(x_t^1, x_t^2\right) < s_c\right) = P\left(\exists t, \left\|x_t^1 - x_t^2\right\| < s_c\right)$$

$$= P\left(\exists t, \left(x_t^1 - x_t^2\right) \in B\left(x_t^2, s_c\right)\right) = P\left(\exists t, c_t^1 \in e\left(c_t^2\right)\right)$$

$$= P\left(c^1 \in \bigcup_t e\left(c_t^2\right)\right)$$

with $x_t^j$ the position of aircraft j, $c_t^j$ the transformed co-ordinate position of aircraft j (j=1, 2), $B(x_t^2, s_c)$ the ball centred on $x_t^2$ and with a radial of $s_c$ and $e(c_t^2)$ the ellipsoid centred on $c_t^2$ □

**Proof lemma 2 :**

$$(2b\Delta y_c)^2 - 4a[c\Delta y_c^{\ 2} - s_c^2] = 0$$

$$\Rightarrow \Delta y_c^2[4b^2 - 4ac] = -4as_c^2$$

$$\Rightarrow \Delta y_c^2 = \frac{as_c^2}{ac - b^2}$$

$$\Rightarrow \Delta y_c = \pm s_c \sqrt{\frac{a}{ac - b^2}} \quad □$$

**Proof lemma 3 :**

Let $\hat{p}_j$ be the exact position of the aircraft j (i.e. without the instruments errors), with j=1,2.

$$d\hat{p}_j = \vec{v}_j dt + s_{hvj} \vec{e}_j dW_{hvj} + s_{hwj} \vec{e}_j dW_{hwj}$$

$$\hat{p}_j(t) = \hat{p}_j(t_i) + \vec{v}_j(t - t_i) + s_{hvj} \vec{e}_1 (W_{hvj}(t) - W_{hvj}(t_i)) + s_{hwj} \vec{e}_j (W_{hwj}(t) - W_{hwj}(t_i))$$

Let the position errors resulting of winds be :

$$\hat{p}_{wj} = e_j(s) \vec{e}_j$$

$$E(\underline{p}_1 \underline{p}_2^{T}) = E\left( \int_0^t e_1(s) \vec{e}_1 ds \int_0^t e_2(s) \vec{e}_2^T ds \right)$$

$$= \vec{e}_1 \vec{e}_2^T E\left( \int_0^t s_{hw1} dW(s) \int_0^t s_{hw2} dW(s) \right)$$

with W a Brownian

then :

$E(\underline{p}_1 \underline{p}_2^{T}) = \pm \vec{e}_1 \vec{e}_2^T s_{hw1} s_{hw2} t$ and $E(\underline{p}_2 \underline{p}_1^{T}) = \pm \vec{e}_2 \vec{e}_1^T s_{hw1} s_{hw2} t$

Thanks to the mean direction of wind, we can know if the two aircraft are deviated in the same direction or in opposite direction □

**Proof lemma 4 :**

The two aircraft can fly in three different ways.

First, the aircraft fly and come close to each other, so the tCPA will be reached at the end of the segment.

Secondly, they move away from each other, so the tCPA is at the beginning of the segment.

Thirdly, they cross each other, and the tCPA is exactly the time of crossing □

**Proof lemma 6 :**

Let $\hat{p}_j$ be the exact position of the aircraft j (i.e. without the instrument errors). j=1,2

$$d\hat{p}_1 = \vec{v}_1 dt + s_{hv1}\vec{e}_1 dW_{hv1} + s_{vv1}\vec{z}\, dW_{vv1} + s_{hw1}\vec{e}_1 dW_{hw1} + s_{vw1}\vec{z}\, dW_{vw1}$$
$$d\hat{p}_2 = \vec{v}_2 dt + s_{hv2}\vec{e}_2 dW_{hv2} + s_{vv2}\vec{z}\, dW_{vv2} + s_{hw2}\vec{e}_2 dW_{hw2} + s_{vw2}\vec{z}\, dW_{vw2}$$

$$\hat{p}_1(t) = \hat{p}_1(t_i) + \vec{v}_1(t-t_i) + s_{hv1}\vec{e}_1(W_{hv1}(t) - W_{hv1}(t_i)) + s_{vv1}\vec{z}(W_{vv1}(t) - W_{vv1}(t_i)) + s_{hw1}\vec{e}_1(W_{hw1}(t) - W_{hw1}(t_i)) + s_{vw1}\vec{z}(W_{vw1}(t) - W_{vw1}(t_i))$$
$$\hat{p}_2(t) = \hat{p}_2(t_i) + \vec{v}_2(t-t_i) + s_{hv2}\vec{e}_2(W_{hv2}(t) - W_{hv2}(t_i)) + s_{vv2}\vec{z}(W_{vv2}(t) - W_{vv2}(t_i)) + s_{hw2}\vec{e}_2(W_{hw2}(t) - W_{hw2}(t_i)) + s_{vw2}\vec{z}(W_{vw2}(t) - W_{vw2}(t_i))$$

Let the position errors resulting of winds be :

$$\hat{p}_{w1} = e_1(s)\vec{e}_1 + x_1(s)\vec{z}$$
$$\hat{p}_{w2} = e_2(s)\vec{e}_2 + x_2(s)\vec{z}$$

with ε a random draw.

$$E(\underline{p}_1\, \underline{p}_2^T) = E\left(\int_0^t e_1(s)\vec{e}_1 + x_1(s)\vec{z}\, ds \int_0^t e_2(s)\vec{e}_2^T + x_2(s)\vec{z}^T ds\right)$$
$$= E\left(\int_0^t s_{hw1}\vec{e}_1 + s_{vw1}\vec{z}\, dW(s) \int_0^t s_{hw2}\vec{e}_2^T + s_{vw2}\vec{z}^T dW(s)\right)$$
$$= \vec{e}_1\vec{e}_2^T E\left(\int_0^t s_{hw1} dW(s) \int_0^t s_{hw2} dW(s)\right) + \vec{e}_1\vec{z}^T E\left(\int_0^t s_{hw1} dW(s) \int_0^t s_{vw2} dW(s)\right)$$
$$+ \vec{z}\,\vec{e}_2^T E\left(\int_0^t s_{hw2} dW(s) \int_0^t s_{vw1} dW(s)\right) + \vec{z}\,\vec{z}^T E\left(\int_0^t s_{vw1} dW(s) \int_0^t s_{vw2} dW(s)\right)$$

therefore :

$E(\underline{p}_1 \underline{p}_2^T) = \pm\left(\vec{e}_1\vec{e}_2^T s_{hw1} s_{hw2} t + \vec{e}_1\vec{z}^T s_{hw1} s_{vw2} t + \vec{z}\,\vec{e}_2^{\,T} s_{hw2} s_{vw1} t + \vec{z}\,\vec{z}^{\,T} s_{vw1} s_{vw2} t\right)$ □

**Proof lemma 7 :**

x(t) is a Ito process (because of its form). Assume that the time random variable $\tau_x$ is an out-going time, function of x(t).

Let $\tau_x$ be $C^2$ for x with bounded derivatives.

The Ito formulate gives us :

$$t(x) = t(x_0) + \int_0^t s\,\frac{\partial t}{\partial x}(x_s)\,dW_s + \int_0^t v\,\frac{\partial t}{\partial x}\,ds + \frac{1}{2}\int_0^t s^2\,\frac{\partial^2 t}{\partial x^2}\,ds$$

We are checking for the mean of $\tau_x$ we can assume that it is the passage time on x with $\tau(x_0)$ equal to zero the passage time on $x_0$.

$$E(t_x) = V(-x)$$

Find a solution to (1) is equivalent to solve the following equation :

$$\lim_{t\to 0}\frac{E(t_x)}{t} = 0$$

The equation corresponding is the Kolmogorov stationary equation. So we obtain on a bounded domain :

$$\begin{cases}\frac{1}{2}s^2\frac{\partial^2 V}{\partial x^2} + v\frac{\partial V}{\partial x} + 1 = 0\\ V(-A) = V(0) = 0\end{cases}$$

the solution to the homogenous system is :

$$V(x) = l\left(e^{\frac{-2v}{s^2}x} - 1\right)$$

and we can find a particular solution :

$$V(x) = -\frac{x}{v}$$

the general solution is :

$$V(x) = l\left(e^{\frac{-2v}{s^2}x} - 1\right) - \frac{x}{v}$$

we can identify the constant with the bound condition :

$$V(x) = l\left(e^{\frac{2vA}{s^2}} - 1\right) + \frac{A}{v} = 0$$

by pass to the infinite we have :

$$l = \frac{-A}{v\left(e^{\frac{2vA}{s^2}} - 1\right)} \xrightarrow[A\to\infty]{} 0 \quad \Rightarrow V(x) = \frac{-x}{v}$$

Note : According to G.R.Grimmet and D.R. Stirzaker[1] [10], an other way to prove the lemma is to show that the moment generating function of t is given by :

$$E\left(e^{y\,t}\right) = \exp\left\{-b\left(\sqrt{a^2 - 2y} + a\right)\right\} \quad for \quad y < \frac{1}{2}a$$

where : t is the random time at witch a standard Brownian process hits the “barrier“ in space-time given by

$$y = a + bt \quad with \quad a < 0\,, b \geq 0 \quad \square$$

**Proof lemma 8 :**

For fixed t :

$$x(t) = x(t_i) + v_i(t - t_i) + s_i(W_i(t - t_i))e_i$$

$$\text{var}(x(t)) = \text{var}(x(t_i)) + (v_i)^2 \text{var}(t_i) + s_i^2(t - E(t_i))e_i e_i^T$$

For $\tau$ the random variable of time :

$$x = x(t) = vt + s\sqrt{t}G = t\left(v + \frac{s\,G}{\sqrt{t}}\right)$$

with G a Gaussian, $E(t) = \frac{x}{v}$ and $s = \sqrt{s_{hv}^2 + s_{hw}^2}$

[1] In exercise 16 p505

$$t = \frac{x}{v + \frac{s}{\sqrt{x/v}} G}$$

$$t = \frac{x}{v} * \frac{1}{1 + aG}$$

with $a = \frac{s}{\sqrt{xv}}$

and near zero, we have :

$$\frac{1}{1+aG} = 1 - aG \left[ + a^2 G^2 + \ldots \right]$$

we chose to stop at the first order, therefore :

$$t = \frac{x}{v} - \frac{x}{v} * \frac{s}{\sqrt{xv}} G$$

$$\mathrm{var}(t) = \mathrm{var}\left(\frac{x}{v}\right) + \mathrm{var}\left(\frac{\sqrt{x}\, s\, G}{\sqrt{v^3}}\right)$$

$$\mathrm{var}(t) = \frac{s_{hp}^2}{v^2} + \frac{s^2}{v^3} \mathrm{var}\left(\sqrt{x}\, G\right)$$

we chose again to stop at first order, then :

$$\mathrm{var}(t) = s_{hp}^2 \frac{1}{v^2} + s^2 \frac{x}{v^3}$$

Note : The proof can be also done by using the moment generating function given in the previous section. □

**Proof lemma 9 :**

Let X(t) and Y(t) be the trajectories of the aircraft beginning at time $t_0$ .

Let V and W be their velocities.

Let $\Delta p_0$ be the position difference at time $t_0$, and $\Delta v$ the constant velocity difference.

$$\vec{X}(t) = \vec{X}_0 + t\vec{V}^1$$
$$\vec{Y}(t) = \vec{Y}_0 + t\vec{V}^2$$

The distance d between the two paths is : $\left\|\vec{X}(t) - \vec{Y}(t)\right\|$

We want to find :

$$t_{lost} = \min_t \{d(t) \le s_c\}$$
$$d(t)^2 = s_c^2$$
$$d^2 = \Delta\vec{p}_0{}^T \Delta\vec{p}_0 + t\Delta\vec{p}_0{}^T \Delta\vec{v} + t\Delta\vec{v}^T \Delta\vec{p}_0 + t^2 \Delta\vec{v}^T \Delta\vec{v} = s_c^2$$
$$\left\|\Delta\vec{p}_0\right\|^2 + 2t\Delta\vec{p}_0^T \Delta\vec{v} + t^2 \left\|\Delta\vec{v}\right\|^2 - s_c^2 = 0$$

by solving this second order equation, we obtain :

$$\Delta = 4\left(\Delta\vec{p}_0^T \Delta\vec{v}\right)^2 - 4\left\|\Delta\vec{v}\right\|^2 \left(\left\|\Delta\vec{p}_0\right\|^2 - s_c^2\right)$$

if $\Delta \ge 0$ and $\Delta v \ne 0$ then :

$$t_{lost} = t_0 + \frac{-2\Delta\vec{p}_0^T \Delta\vec{v} - \sqrt{4\left(\Delta\vec{p}_0^T \Delta\vec{v}\right)^2 - 4\left\|\Delta\vec{v}\right\|^2 \left(\left\|\Delta\vec{p}_0\right\|^2 - s_c^2\right)}}{2\left\|\Delta\vec{v}\right\|^2}$$

else, if $\Delta < 0$ and $\Delta v \ne 0$ there is no solution, the separation is never lost

if $\Delta v = 0$ the separation between the two aircraft is constant □

**Proof property 1 :**

That corresponds to the case where $\Delta < 0$

$$\left(\Delta\vec{p}_0^T \Delta\vec{v}\right)^2 - \left\|\Delta\vec{v}\right\|^2 \left(\left\|\Delta\vec{p}_0\right\|^2 - s_c^2\right) < 0$$
$$\left\|\Delta\vec{p}_0\right\|^2 \left\|\Delta\vec{v}\right\|^2 \cos\left(\Delta\vec{p}_0, \Delta\vec{v}\right) - \left\|\Delta\vec{v}\right\|^2 \left(\left\|\Delta\vec{p}_0\right\|^2 - s_c^2\right) < 0$$
$$\frac{s_c^2}{\left\|\Delta\vec{p}_0\right\|^2} < 1 - \cos^2\left(\Delta\vec{p}_0, \Delta\vec{v}\right)$$
$$\frac{s_c^2}{\left\|\Delta\vec{p}_0\right\|^2} < \sin^2\left(\Delta\vec{p}_0, \Delta\vec{v}\right) \quad □$$

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# APPENDIX B : TIME AS A RANDOM VARIABLE

Usually conflict detection methods consider the time as a deterministic parameter. But the reality of ATC contradicts this assumption. Indeed, if the two aircraft fly with FMS, then they have higher delay uncertainties than position uncertainties.

That is to say , the aircraft have the ability to follow the nominal trajectories with a high precision and to turn exactly at the required point. But they can not really be punctual (except for the new FMS 4D), for several reasons e.g. the aircraft can be delayed due to unexpected changes in wind.

So to consider the time as a random variable and then to determine its properties, will help to model the navigation with FMS.

This will be a preamble for a future study where the aircraft will turn exactly at the required points.

## 1. THE MEAN OF TIME

The expression of the time in function of the others known parameters is needed.

For a fixed time *t* we have : $x(t) = x(t_i) + v_i(t - t_i) + s_i(W_i(t - t_i))e_i$

with $\vec{e}_i$ the normalised direction vector of the aircraft and $Wi$ the Brownian noise on the segment $[x_i ; x_{i+1}[$ .

$x(t_i)$ is a random variable, but this fact will be considered as negligible because, the instrument error of position is small.

*Note : The Brownian movement is usually used by physicists to model the trajectory of a small particle. In a liquid, this particle makes uncoordinated movements as a result of the perpetual shocks that it experiences from collisions with similar neighbouring particles.*

*A process is said to be Brownian if : it is continuous, its growths are independent and stationary, and if each component of the process is a centred Gaussian (that is to say, with its mean equal to zero).*

*It has been chosen to represent the noise on the aircraft position by a Brownian because of all these characteristics.*

It is always true to say that (because of the conditional mean properties) :

$$E(x(t)/t_i) = x(t_i) + v_i(t - t_i)$$

$$E(x(t)) = x(t_i) + v_i(t - E(t_i))$$

furthermore by assumption (because by definition $t_{i+1}$ is the first time at which the position $x_{i+1}$ is reached)

$x(t_{i+1}) = x_{i+1}$

then the mean time can be verified by:

$x_{i+1} = E(x(t_{i+1})) = x_i + v_i E(t_{i+1} - t_i) + s_i e_i E(W_i(t_{i+1} - t_i))$

The following problem is obtained :

$E(W(t_x)) = 0 = E(E(W(t_x)/t_x)) \quad \forall t_x$ (1)

with $t_x$ a time random variable corresponding to the time when the aircraft fly over the point x.

That is to say :

$dx = vdt + s\ dW$

$x(t) = vt + s\ W(t)$

with *W* a Brownian and *dW* a white noise.

*Note : To simplify the comprehension of this expression, a white noise can be seen as the derivative of a Brownian, even if it is not mathematically correct.*

LEMMA 7 :

> The solution of (1) is : $E(t_x) = \frac{x}{v}$

*Proof : Appendix A*

It is noted that as expected, the mean of the time random variable is equal to its nominal value.

## 2. THE VARIANCE OF TIME

There is a time difference between the estimated duration of a flight and the real duration of this one. It is helpful to quantify this difference.

The variance of time gives an interval during which a wanted position *x* will be reached, almost certainly.

LEMMA 8 :

> The variance of time can be approximated by :
>
> $\mathrm{var}(t) = \frac{s_{hp}^2}{v^2} + \left(s_{hv}^2 + s_{hw}^2\right)\frac{x}{v^3}$

*Proof : Appendix A*

## 3. TIME WHEN SEPARATION IS LOST

When a conflict occurs, it is interesting to look at the predicted time when the separation required is lost. This information will help later to solve the conflict.

The separation is lost as soon as the conflict probability exceeds a fixed value, on the first segment of route where it occurs.

The previous section showed that the mean time is equal to its nominal value.

LEMMA 9 :

> Assumptions : velocities are constant, $\Delta v$ is different from zero, a conflict occurs.
>
> Then, the predicted time at which the required separation is lost, and the mean of this time, is :
>
> $$E(t_{lost}) = t_{lost} = t_0 + \frac{-2\Delta\vec{p}_0^T\Delta\vec{v} - \sqrt{4\left(\Delta\vec{p}_0^T\Delta\vec{v}\right)^2 - 4\|\Delta\vec{v}\|^2\left(\|\Delta\vec{p}_0\|^2 - s_c^2\right)}}{2\|\Delta\vec{v}\|^2}$$
>
> where $\Delta p_0$ is the position difference at time $t_0$, and $\Delta v$ is the constant velocity difference.

*Proof : Appendix A*

PROPERTY 1 :

> No conflict will occur during the flights if : $\frac{s_c^2}{\|\Delta\vec{p}_0\|^2} < \sin^2\left(\Delta\vec{p}_0, \Delta\vec{v}\right)$

*Proof : Appendix A*

*Note : when $\left(\Delta\vec{p}_0, \Delta\vec{v}\right)$ makes a right angle then a conflict will occur.*

## 4. THE POTENTIAL PERIOD OF CONFLICT

It was shown before that the predicted time of closest approach is tCPA, with :

tCPA = $t_0 - \frac{\Delta p_0^T \Delta v}{\Delta v^T \Delta v}$ = $E(\tau\text{CPA})$

Now it is required to determine the period of potential conflict for both aircraft. The predicted time when the required separation is lost is given by:

$$t_{lost} = E(t_{lost}) = t_0 + \frac{-\Delta\vec{p}_0^T\Delta\vec{v} - \sqrt{\left(\Delta\vec{p}_0^T\Delta\vec{v}\right)^2 - \left\|\Delta\vec{v}\right\|^2\left(\left\|\Delta\vec{p}_0\right\|^2 - s_c^2\right)}}{\left\|\Delta\vec{v}\right\|^2}$$

In this way, the predicted time when separation is restored can be found:

$$t_{return} = E(t_{return}) = t_0 + \frac{-2\Delta\vec{p}_0^T\Delta\vec{v} + \sqrt{4\left(\Delta\vec{p}_0^T\Delta\vec{v}\right)^2 - 4\left\|\Delta\vec{v}\right\|^2\left(\left\|\Delta\vec{p}_0\right\|^2 - s_c^2\right)}}{2\left\|\Delta\vec{v}\right\|^2}$$

The predicted potential period of conflict is the interval between the time when separation is lost and the time when it is restored.

Thanks to the variance of time, found in the previous section (section 3.2), the potential period of conflict that takes navigation errors into account, can be verified.

It is known that : $\mathrm{var}(t) = \frac{s_{hp}^2}{v^2} + \left(s_{hv}^2 + s_{hw}^2\right)\frac{x}{v^3}$

So for each aircraft, the potential period of conflict is :

$$\left[t_{lost} - \sqrt{\mathrm{var}(t_{lost})}\ ;\ t_{return} + \sqrt{\mathrm{var}(t_{return})}\right]$$

## 5. COMMENT

This study of time as a random variable will not be used for the detection of conflict in this report. Because it needs its own study and this one is not achieved. But it could be a useful insight into understanding aircraft delays. Furthermore it shows that alternative approaches, are possible. This will be studied in future work.

# APPENDIX C : GLOSSARY

Extracted from [9]

**Accuracy.** A measure of the difference between the reported or measured aircraft position as compared to the true position. Accuracy is usually defined in statistical terms of either a mean and a variation about the mean as defined by the standard deviation (sigma) or a root mean square (RMS) value from the mean. (RTCA SC-186, August 1997)

**Advisory.** A message given to the pilot containing information relevant to collision avoidance. This term is synonymous with alert. (RTCA SC-147, December 1997)

**Airborne Collision.** This occurs when two aircraft in flight come into contact. (SICASP WG2 WP2/641, June 1997)

**Airborne Collision Avoidance.** An unplanned manoeuvre to avoid a collision. (RTCA SC-186, August 1997)

**Airborne Collision Avoidance System.** An aircraft system based on secondary surveillance radar (SSR) transponder signals, which operates independently of ground-based equipment to provide advice to the pilot on potential collisions with aircraft that are equipped with SSR transponders. (SICASP WG2 WP2/724, October 1998)

**Airborne Separation Assurance System.** The equipment, communications, protocols, airborne surveillance and other aircraft state data, flight crew and ATC procedures which enables the pilot to exercise responsibility, in agreed and appropriate circumstances, for separation of his aircraft from one or more aircraft. (SICASP WG2 WP2/724, October 1998)

**Air Traffic Control.** The tactical safety separation service whose function is to prevent collisions between aircraft and between aircraft, terrain and obstructions. (Kayton, M. and Fried W.R., 1997)

**Air Traffic Management.** The purpose of Air Traffic Management (ATM) is safe, efficient and expeditious movement of aircraft in the airspace. It comprises two principal processes: Air Traffic Control and Traffic Flow Management. (Kayton, M. and Fried W.R., 1997)

**Alarm.** An aural signal to the pilot that recommends immediate attention to the displays. (RTCA SC-147, December 1997)

**Alert.** Synonymous with advisory. (RTCA SC-147, December 1997)

**ALIM.** The desired amount of vertical distance when selecting a resolution manoeuvre.

**Area Navigation.** The on-board navigation system determines the position of the aircraft and a navigation computer, often embedded in the flight management system, carries out the necessary course computations for reaching the next way-point. Aircraft Area Navigation capability are not constrained to travel directly toward or away from ground-based VORs as are aircraft flying along airways. Position determination may be based on station-referenced navigation signals, such as VOR, DME, LORAN-C, and GNSS or a self-contained capability, such as an inertial reference unit. (Kayton, M. and Fried W.R., 1997)

**Automatic Dependent Surveillance.** A function that automatically transmits, via a data link, position data derived from the on-board navigation system. (Kayton, M. and Fried W.R., 1997)

**Automatic Dependent Surveillance-Broadcast.** An application, related to surveillance, transmitting parameters, such as position, track and groundspeed via a broadcast mode datalink, and at specified intervals, for utilisation by any air and/or ground users requiring it. ADS-B is a datalink application. (SICASP WG2 WP2/724, October 1998)

**Availability.** The fraction of time that the system is usable for its function (e.g., navigation or surveillance).

**Bearing.** The angle of the target aircraft in the horizontal plane, measured clockwise from the longitudinal axis of the own aircraft. (FAA, March 1990)

**Broadcast.** Unsolicited transmission to a non-specific destination. (RTCA SC-147, December 1997)

**Call sign.** The radiotelephony code assigned to an aircraft for voice communication purposes. This term is sometimes used interchangeably with ‘flight identification’ or ‘flight ID’. For general aviation aircraft, the aircraft call sign is normally its national registration number; for airline and commuter aircraft, it is usually comprised of the company name and flight number (and therefore not linked to a particular airframe); and for the military, it usually consists of numbers and code words with special significance for the operation being conducted. (RTCA SC-186, August 1997)

**Closest Point of Approach.** The occurrence of minimum range between the own and target aircraft. Thus, range at closest approach is the smallest range between the two aircraft and time of closest approach is the time at which this occurs. (RTCA SC-147, December 1997)

**Cockpit Display of Traffic Information.** A function which provides the flight-crew with surveillance information about other aircraft, including their position. The information may be presented on a dedicated multi-function display (MFD), or be processed for presentation on existing cockpit flight displays. Traffic information for the CDTI function may be obtained from one or multiple sources (including ADS-B, TCAS and TIS) and it may be used for a variety of purposes. Requirements for CDTI information will be based on intended use of the data (i.e., application). (RTCA SC-186, May 1998)

**Conflict.** Predicted converging of aircraft in space and time, which constitutes a violation of a given set of separation minima. (SICASP WG2 WP2/724, October 1998)

**Conflict Detection.** The process of projecting an aircraft's trajectory to determine whether it is probable that the applicable separation minimum will not be maintained between the aircraft and either 1, another aircraft or vehicle, 2, a given airspace, or 3, ground terrain. The level of uncertainty in the projection is reduced with increased knowledge about the situation, including aircraft capabilities, flight plan, short-term intent information, etc. (RTCA SC-186, August 1997)

**Conflict Management.** Process of detecting and resolving conflicts. (RTCA SC-186, August 1997)

**Conflict Resolution.** The process of identifying a manoeuvre or set of manoeuvres that, when followed, do not cause a conflict or reduce the likelihood of conflict between an aircraft and either 1, another aircraft or vehicle, 2, a given airspace, or 3, ground terrain. Manoeuvres may be given to multiple aircraft to fully resolve the conflict. (RTCA SC-186, August 1997)

**Continuity.** The ability of the total system to perform its function without interruption during the intended operation. More specifically, continuity is the probability that the system will be available for the duration of a phase of operation, presuming that the system was available at the beginning of that phase of operation. (Kayton, M. and Fried W.R., 1997)

**Co-operative surveillance.** Surveillance technique, which requires for an aircraft to be seen that it has simple transponding equipment on board.

**Co-ordination.** The process by which two TCAS-equipped aircraft select compatible (non-conflicting) Resolution Advisories (RTCA SC-147, December 1997)

**Corrective Advisory.** A resolution advisory that instructs the pilot to deviate from current vertical rate, e.g., 'Don't Climb' when the aircraft is climbing.

**Correlation.** The process of determining whether two or more reports refer to a single aircraft. (RTCA SC-186, May 1998)

**Data fusion.** A function that may include the following calculations: (1) time extrapolation and transformation of individual input reports into display co-ordinates at the next CDTI update time, (2) pair-wise correlation of input reports from different surveillance sources in order to display one unique track file per aircraft, and (3) selection or blending of the best report data from correlated input report pairs for display output. If only one source of surveillance data is available, then the calculations in (2) and (3) can be omitted. (RTCA SC-

186, May 1998) This definition is drawn up specifically for the CDTI-application. The basic idea of data fusion is the same for other application.

**Datalink.** Digital data communication for air-to-air, air-to-ground or ground-to-ground applications.

**Dependent Surveillance.** Surveillance technique of which the operation and quality depend upon the performance of the aircraft's navigation system. (Kayton, M. and Fried W.R., 1997)

**Distance Modification.** Safety factor incorporated in range measurements to account for possible accelerations by the intruder. The value of distance modification varies with the sensitivity level for this own intruder set. The value is chosen such that a sustained acceleration of g/3 will produce this displacement in range threshold time. (RTCA SC-147, December 1997)

**Flight ID.** Synonymous for Call Sign.

**Free Flight.** A safe and efficient flight operating capability under Instrument Flight Rules (IFR) in which the operators have the freedom to select their path and speed in real time. (RTCA SC-186, August 1997)

**Free Flight Airspace.** Free Flight Airspace will consist of airspace (defined by vertical, lateral and time boundaries) where suitably equipped aircraft will be able to fly user-preferred 3-D or 4-D routings. Responsibility for the separation assurance from other aircraft operating in the same airspace will rest with the aircraft in almost all circumstances, although some responsibility can be undertaken by ground-based ATM (emergencies) or delegated to other organisations (principally the military). (EUROCONTROL, January 1999)

**Global Navigation Satellite System.** A world-wide position, velocity, and time determination system, that includes one or more satellite constellations, receivers, and system integrity monitoring, augmented as necessary to support the required navigation performance for the actual phase of operation. (RTCA SC-186, May 1998)

**Horizontal Miss Distance.** The horizontal range between two aircraft at the point of closest approach. (RTCA SC-147, December 1997)

**Inertial Navigation System.** An on-board system that determines the own ship's position and velocity by measuring its acceleration and processing the acceleration information in a computer.

**Integrity.** The ability of a system to provide timely warnings to aircraft when its errors are excessive. (Kayton, M. and Fried W.R., 1997)

**Intent.** Information describing an aircraft's future trajectory. (SICASP WG2 WP2/724, October 1998)

**Intruder.** A transponder-equipped aircraft within the surveillance range of TCAS for which TCAS has an established track. (RTCA SC-147, December 1997)

**Managed Airspace.** Managed Airspace in 2015 will consist of airspace (defined by vertical, lateral and time boundaries) that will be needed to support en-route operations within which the control of aircraft is the responsibility of the ground ATM organisation. Traffic structuring, in the form of 2-D and 3-D route networks will be used in the busiest areas at peak times to enhance capacity, to organise traffic flows and to reduce the incidence of conflicts for en-route and TMA operations. In other areas and outside peak times in the busiest areas, MAS will support the operation of aircraft using user-preferred trajectories. (EUROCONTROL, January 1999)

**Mode A.** A type of ATCRBS transmissions which requests (via Mode A interrogations) or reports (via Mode A replies) aircraft identity information. (RTCA SC-147, December 1997)

**Mode C.** A type of ATCRBS transmissions which requests (via Mode C interrogations) or reports (via Mode C replies) aircraft altitude information. ATCRBS transponders which do not have Mode C capability respond to Mode C interrogations with standard ATCRBS framing pulses but no altitude information. (RTCA SC-147, December 1997)

**Mode S.** A type of SSR transmission which contains a unique 24-bits discrete address, thus allowing interrogations to be addressed to individual aircraft. Mode S transmissions can be short (56 bits) or long (112 bits), with long transmissions containing a 56-bit message field. (RTCA SC-147, December 1997)

**Navigation.** The on-board determination of the position and velocity of a moving vehicle. (Kayton, M. and Fried W.R., 1997)

**Navigation Uncertainty Category.** Uncertainty categories for the state vector navigation variables are characterised by a NUC data set provided in the ADS-B sending system. The NUC includes both position and velocity uncertainties. (RTCA SC-186, August 1997)

**Near mid-air collision.** Two aircraft simultaneously coming within 100 ft vertically and 500 ft horizontally. (RTCA SC-147, December 1997)

**Non-synchronous garble.** Reply pulses received from a transponder that is being interrogated from some other source. (FAA, March 1990)

**Potential threat.** An intruder that has passed the Potential Threat classification criteria for a TA and does not meet the Threat Classification criteria for an RA. (RTCA SC-147, December 1997)

**Preventive Advisory.** A resolution advisory that instructs the pilot to avoid certain deviations from current vertical rate, as for example ‘Don’t Climb’ when the aircraft is level. (RTCA SC-147, December 1997)

**Primary means of surveillance.** A preferred means (when other means are available) of obtaining surveillance data for aircraft separation and avoidance of obstacles. (RTCA SC-186, August 1997)

**Primary Surveillance Radar.** A radar system in which determines target range, azimuth, and, in some cases, elevation from the signal reflected back by the aircraft skin.

**Proximate traffic.** Nearby aircraft within 1200 feet and 6 nautical miles which do not meet either the threat or the potential threat classification criteria. (RTCA SC-147, December 1997)

**Report.** A message containing surveillance data on a target aircraft. (RTCA SC-147, December 1997)

**Resolution Advisory.** A display indication given to the pilot recommending a manoeuvre to either increase or maintain the existing vertical separation relative to and intruding aircraft. (RTCA SC-147, December 1997)

**Resolution Manoeuvre.** Manoeuvre resulting from compliance with a Resolution Advisory. (RTCA SC-147, December 1997)

**Secondary Surveillance Radar.** A radar system in which the return signal is radiated from a transmitter on board the target. (RTCA SC-147, December 1997)

**Sensitivity level.** A set of parameters used to specify the size of the protected volume around the TCAS-equipped aircraft for (potential) threat detection. The size of the protected volume and hence the Sensitivity Level varies with altitude. (RTCA SC-147, December 1997)

**Separation.** Separation exists between two or more aircraft when their positions and velocities are in accordance with standards or procedures that have been determined to be appropriate for the operations in which the aircraft are engaged. (SICASP WG2 WP2/641, June 1997)

**Separation assurance.** The process by which assurance is provided that separation is maintained. (SICASP WG2 WP2/641, June 1997)

**Situational awareness.** The knowledge of the position and other information such as the identity, status and the intentions of the other aircraft with respect to the own aircraft's trajectory. (SICASP WG2 WP2/724, October 1998)

**Squitter.** Spontaneous transmission generated by Mode S transponders. (RTCA SC-147, December 1997)

**Station keeping.** Station keeping provides the capability for a pilot to maintain an aircraft's position relative to the designated aircraft. For example, an aircraft taxiing behind another aircraft can be cleared to follow and maintain separation on a lead aircraft. Station keeping can be used to maintain a given (or variable) separation. An aircraft that is equipped with an ADS-B receiver could be cleared to follow an FMS or GNSS equipped aircraft on a GNSS/FMS/RNP approach to an airport. An aircraft doing station keeping would be required to have, as a minimum, some type of CDTI. (RTCA SC-186, August 1997)

**Surveillance.** The out-board determination of the position and velocity of a moving vehicle. (Kayton, M. and Fried W.R., 1997)

**Synchronous garble.** An overlap of the reply pulses received from two or more transponders answering the same interrogation. (FAA, March 1990)

**Target.** An aircraft within the surveillance range.

**Threat.** A target that has satisfied the threat detection logic and thus requires a resolution advisory. (RTCA SC-147, December 1997)

**Track.** Estimated position and velocity of a single aircraft based on correlated surveillance data reports. (RTCA SC-147, December 1997)

**Traffic Advisory.** Information given to the pilot pertaining to the position of another aircraft in the immediate vicinity. The information contains no suggested manoeuvre. (RTCA SC-147, December 1997)

**Traffic Flow Management.** The process that allocates traffic flows to scarce capacity resources (e.g., it meters arrival at capacity constrained airports). (Kayton, M. and Fried W.R., 1997)

**Traffic Information Service Broadcast.** The dissemination of aircraft position reports as collected by the ground-based radar surveillance system to participating users. (RTCA SC-186, August 1997)

**Trajectory Change Point.** The Trajectory Change Points provide tactical information specifying space/time points at which the current trajectory of the vehicle will change. (RTCA SC-186, August 1997)

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# APPENDIX D : ADS-B

Extracted from [9]

Automatic Dependent Surveillance-Broadcast (ADS-B) [3], a new means to provide air-to-air and air-to-ground communication allows the employment of ASAS techniques.

## 1 Definition and principle

'ADS-B is a function on an aircraft or surface vehicle that periodically broadcasts its state vector (horizontal and vertical position, horizontal and vertical velocity) and other information. ADS-B is automatic because no external stimulus is required to elicit a transmission. It is dependent because it relies on on-board navigation sources and on-board broadcast transmission systems to provide surveillance information to other users. The aircraft or vehicle originating the broadcast need have no knowledge of which users are receiving its broadcast. Any user, either aircraft or ground-based, within range of this broadcast, may choose to receive and process ADS-B surveillance information.' (RTCA SC-186, August 1997)

Based on the intended use, the ADS-B subsystem may be interactive, broadcast only or receive only. Different implementation methods for ADS-B are under investigation. These are not considered in this report.

## 2 Expected benefits

ADS-B is intended to be capable of becoming a primary means of surveillance supporting a variety of air traffic applications like a cockpit display of traffic information, aircraft-based collision avoidance, conflict management and airspace deconfliction, simultaneous approaches and others.

The benefits can be divided into three main categories:

- Pair-wise benefits (single aircraft to single aircraft)

To realise voluntary equipage with ADS-B, benefits should be available to users equipping before there is substantial aircraft equipage or ground infrastructure. These early benefits will result from the use of procedures that only require an aircraft to have situational awareness of another specific aircraft. The benefits in this category include capabilities such as in-trail passing manoeuvres, station keeping and supporting technology for enhanced collision avoidance systems between equipped aircraft.

- User benefits based on ground infrastructure improvements

ADS-B provides more accurate position information than either Primary or Secondary Surveillance Radar (PSR, respectively SSR). In addition, ADS-B provides other types of information, including accurate velocity and aircraft intent.

Individual aircraft equipped with ADS-B may realise additional benefits once a ground infrastructure has been deployed. Capabilities that may be enabled are:

- surveillance services in airspace with limited or no radar coverage,
- improved airport surface operations,
- reduced and more flexible separation requirements,
- improved ATS ground-based conflict detection operation, and
- optimisation of the traffic flow.

An ADS-B equipped aircraft fitted with the necessary data assimilation units, could receive and use data coming from the Traffic Information Service-Broadcast [2] (TIS-B).

- Full population benefits

At some time, the aircraft population in high-density terminal areas or aircraft operating in high altitude airspace will be fully equipped. For this airspace, there are a number of benefits that include:

- enhanced situational awareness,
- improved aircraft- and ground-based conflict management, and
  potential for reduced infrastructure costs.

To conclude, ADS-B supports increased airspace and airport capacity while maintaining or even improving the high level of safety in a cost-effective manner.

[2] Traffic Information Services-Broadcast is another potential application of broadcast services, disseminating aircraft position reports as collected by the ground-based radar surveillance system to participating users for cockpit display of traffic information. (RTCA SC-186, August 1997)

# 3 Aircraft equipment

The aircraft equipment for ADS-B includes a message generation function, a transmission function, a receiver with message receipt and report assembly processing function (optional in some implementations) and a number of interfaces. This is illustrated in Figure 1.

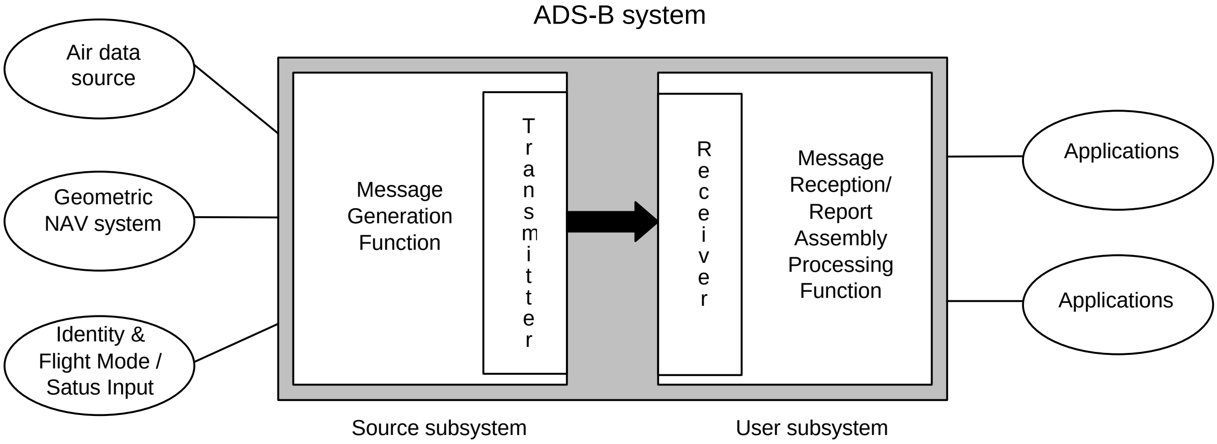


Figure 1: ADS-B block diagram

For participation in an ADS-B based application, common equipage is required.

The ADS-B system is provided with the necessary data to generate the messages conveying identification, state vector, status and intent information by different onboard data sources:

- Air data source – The system interfaces with the onboard barometric altitude source. The input includes an indication of data validity and an indication of 100 or 25 ft resolution provided by the source. If available, altitude rate and airspeed are put into the system. The airspeed is either IAS or TAS with a discrete to indicate data type.

- Geometric navigation system – There is an interface with the onboard source of geometric position and velocity navigation data.[3] The navigation data is accompanied with the current Navigation Uncertainty Category (NUC) of the source. The data includes logic to indicate validity and no computed data. Back up sources have to be available in the event of loss of the normal primary source.

[3] One can assume that the geometric navigation system will have Area Navigation (RNAV) capability. Aircraft with RNAV are not constrained to travel directly toward or away from ground-based radio beacons as are aircraft flying along airways. Position determination may be based on different systems. It is assumed to be based on a Global Navigation Satellite System (GNSS) for most of the time, potentially validated with the Inertial Navigation System (INS). In case of failure of both systems, data can be derived from other station-referenced navigation such as VOR, DME, Loran C, etc.

- Identification and Flight Mode/Status Data Input Devices – The system interfaces with the onboard data entry mechanisms such as flight deck keyboards and selectors to acquire the relevant information.

# 4 Information transfer

The ADS-B system transmits messages containing the information specified in the following subparagraphs.

The triggering of messages may be event driven (e.g., entering the approach area, encountering turbulence, etc.) or possibly may be turned on by crew action (e.g., emergency, priority handling, etc.). ADS-B applications have to accommodate missing and/or temporary interruption of required data elements.

The ADS-B system accepts own-ship source data from each aircraft interactive participant, aircraft broadcast-only participant and ground broadcast-only participant, and makes it available to each of the other interactive participants as well as the receive-only ground sites. Interactive ground systems may also exist in some ADS-B systems.

## 4.1 Identification

The basic identification information conveyed by ADS-B includes the following elements:

- call sign,
- address, to correlate all messages transmitted from the aircraft and to differentiate them from messages from other aircraft in the operational domain, and
- category, e.g., light, medium and heavy aircraft, high performance, glider/sailplane, surface vehicle, etc.

## 4.2 State vector

The reported state vector for an aircraft includes the three-dimensional position and velocity referenced to a world-wide reference system. Some elements of the aircraft's acceleration may be included.

Position

- horizontal latitude and longitude,

- barometric pressure altitude, referenced to standard temperature and pressure, and

- geometric height, defined as the minimum altitude above or below a plane tangent to the earth

Velocity

Horizontal velocity components are defined as the north-south and east-west velocity in knots. Altitude rate is designated as climbing or descending.

Acceleration

Airborne turn rate is designated as turning right or left. Taxi speed change indicates increasing or decreasing speeds while taxiing.

Time of applicability

All reports contain the time of applicability. It indicates the time at which the reported values were valid.

Navigation Uncertainty Category (NUC)

Uncertainty categories for the state vector navigation variables are characterised by a NUC data set provided in the ADS-B sending system. The NUC includes both position and velocity uncertainties. Position components of a NUC are represented as NUCp. Velocity components are represented as NUCr.

Table 1 : Navigation Uncertainly Categories – Position

| NUCp | Horizontal Error (95%) | Vertical Error (95%) |
|---|---|---|
| 0 | Unknown | Unknown |
| 1 | < 10 nmi | Baro Alt |
| 2 | < 5 nmi | Baro Alt |
| 3 | < 1 nmi | Baro Alt |
| 4 | < 0.5 nmi | Baro Alt |
| 5 | < 0.25 nmi | Baro Alt |
| 6 | < 0.1 nmi | Baro Alt |
| 7 | < 0.05 nmi | Baro Alt |
| 8 | < 10m | < 15 m |
| 9 | < 3m | < 4 m |

Table 2 : Navigation Uncertainly Categories – Velocity

| NUCr | Horizontal Velocity Error (95%) | Vertical Velocity Error (95%) |
|---|---|---|
| 0 | Unknown | Unknown |
| 1 | < 10 m/s | < 50 fps |
| 2 | < 3 m/s | <15 fps |
| 3 | < 1 m/s | < 5 fps |
| 4 | < 0.3 m/s | < 1.5 fps |

### 4.3 Status and intent information

Status and intent information is used to support ATS and aircraft-to-aircraft applications.

*Emergency/Priority status,* e.g., no emergency, minimum fuel, no communications, general emergency, etc.

Intent information

The ADS-B system has the capability to exchange Trajectory Change Point (TCP) and Trajectory Change Point + 1 (TCP+1) data. The TCP defines a point in three dimensional airspace where the current operational trajectory is planned to change (e.g., top of descent, reach climb altitude, etc.). In addition, the estimated remaining flight time to the TCP and the type of path to be followed to the TCP (e.g., heading, procedure turn, holding pattern, etc.) are given.

The intent information is used for applications on the receiving aircraft or ATS to support stable separation predictions for long look-ahead times, in monitoring required operational separations and to re-plan flight paths when necessary to mitigate detected conflicts while minimising deviations from planned flight trajectories.

ADS-B transmission will indicate the ability of the transmitting participant to engage in path monitoring and/or deconflicting operations. The transmitting aircraft also indicates its ability to use intent information received from other participants.

### 4.4 ADS-B system performance

The following information is derived from the Minimum Aviation System Performance Standards for ADS-B (RTCA SC-186, August 1997).

Accuracy

The accuracy of the state vector information affects its utility for surveillance applications. Factors that affect state vector accuracy include:

- errors in the navigational system,

- errors in the ADS-B reporting system's
- errors in the time of applicability, and
- errors introduced by processing.

Availability

Where the ADS-B system is used as a supplemental means of surveillance, it is expected to be available with a probability of at least 0.95 for all operations, independent of the availability of appropriate inputs to the ADS-B system.

Where the ADS-B system is used as a primary means of surveillance, the system is expected to be available with a probability of at least 0.999 for all air-to-air operations. In addition, a supplemental surveillance system, independent of the navigation system, is expected to be available. The overall surveillance system needs to satisfy fail-safe operation of navigation and surveillance. This will enable ATS to provide an independent means of guidance to aircraft losing all navigation capability. The expected availability of the total surveillance system is at least 0.99999, independent of navigation system availability.

Continuity

The probability that the ADS-B system is unavailable during an operation, presuming that the system was available at the start of the operation, is no more than $2 \times 10^{-4}$ per hour of flight.

Integrity

The maximum probability that the ADS-B system, when supplied with correct source data, provides an output report that has undetected errors is $10^{-5}$.

To conclude, ADS-B is expected to provide very high quality data.