A Review of Airport Runway Scheduling

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Introduction

The scheduling of landings and the scheduling of take-offs at an airport runway are problems with different characteristics. We deal with each separately.

One of the interesting features of runway scheduling is the presence of wake vortices (a rotating mass of air) caused by aircraft in flight. Safety reasons dictate certain minimum separations between successive aircraft using the runway. These separations depend on the sizes of the leading and following aircraft and are normally specified as minimum time gaps.
Most of Europe classifies aircraft as large, medium or small for the purposes of specifying minimum separations. However, the UK has five weight categories: large, upper medium, lower medium, small and light.
Scheduling Aircraft Landing

The most basic problem has a single runway dedicated to landing aircraft. Associated with each aircraft \( i \), we have:

\([E_i,L_i]\): a time window during which the aircraft must land, where \( E_i \) is a release date and \( L_i \) is a deadline

\( T_i \): a target (preferred) landing time, where \( E_i \leq T_i \leq L_i \)

Also, we define:

\( S_{uv} \): the minimum separation time between aircraft of weight categories \( u \) and \( v \) (where the triangle inequality is assumed)

The may also be precedence constraints possibly reflecting airline preferences, or to avoid potential conflicts if two aircraft are following the same jet route so that overtaking must be avoided.
Scheduling Aircraft Landing

Safety considerations may dictate that the landing sequence must be close to a first-come-first-served (FCFS): large deviations from FCFS can lead to an excessive workload for air traffic controllers. A position shift constraint can be imposed: no landing can deviate by more than $k$ positions relative to the FCFS sequence.

It is possible to represent a solution as a sequence of landings. Unless there are penalties for landing before the target time, it is straightforward to compute actual earliest landing times for a given sequence. Possible objective functions are to minimize the time by which all aircraft land (makespan) or to minimize the total penalty (total weighted tardiness) from exceeding target landing times.
Psaraftis (1980) considers problem of sequencing $W$ weight categories of aircraft to minimize the makespan, where aircraft having the same weight category are regarded as identical. Using state variables $(w, n_1, \ldots, n_W)$, where $w$ is the weight category of the last aircraft to land, and $n_i$ aircraft of category $i$ have landed, he develops an $O(W^2 n^W)$ dynamic programming algorithm.

The algorithm can be adapted to handle position shift constraints and precedence constraints. Since $W$ is typically between 3 and 5, it represents a practical solution method.
Dynamic Programming for Aircraft Landing

Trivizas (1998) proposes an $O(n2^k)$ dynamic programming algorithm for the makespan problem with position shift constraints. Since $k$ is typically about 3, the algorithm is practical, but apparently cannot deal with precedence constraints.

Balakrishnan and Chandran (2006) develop an alternative dynamic programming algorithm that can handle precedence constraints and time windows for landing times. Their states at stage $j$ comprise a sequence of landings for positions $j, ..., j+2k$. The algorithm requires $O(n(2k+1)^{2k+2})$ time.
Brentnall (2006) generalizes the dynamic programming approach of Psaraftis by allowing jobs within the same weight category to have different time windows for landing, but with agreeable earliest and latest landing times. The makespan problem is then solved in $O(W^2n^W)$ time. Also, the total tardiness problems with target landing times equal to earliest landing times is solved in $O(W^2n^{W^2+W+1})$ time.
Brentnall (2006) also considers the problem of aircraft forming $S$ stacks, where aircraft within a stack must land according to their position in the stack. Dynamic programming algorithms for general objective functions are derived with a time complexity of $O(S^2 n^{S+W})$. 
Local Search Methods for Aircraft Landing

Ernst, Krishnamoorthy and Storer (1999) consider a problem with time windows for landing, and linear earliness and tardiness penalties for landing before and after the target landing time, respectively. They consider the single and multiple runway problem.

Given a landing sequence, they develop an efficient methods for determining optimal landing times. The use a “problem space” genetic algorithm to search for an optimal set of parameters to use in a heuristic that finds a landing order.

They also derive a branch and bound algorithm that gives an optimal solution.
Local Search Methods for Aircraft Landing

Beasley, Sonander and Havelock (2001) consider aircraft landings at Heathrow. They consider time windows for landing, and an objective function comprising the sum of squared tardiness minus the sum of squared earliness (evaluated with respect to the target time).

They propose a genetic algorithm where solutions are represented by landing times. A penalty function for landing times that violate minimum separation constraints is used to produce feasible solutions.
Local Search Methods for Aircraft Landing

Pinol and Beasley (2006) provide a follow up to the work of Beasley, Sonander and Havelock (1999). The consider multiple runways and both the linear and quadratic objective functions introduced above. Scatter search and bionomic are proposed. Solutions are represented by landing times and runway allocations.
Scheduling Aircraft Departures

The most basic problem has a single runway dedicated to departing aircraft. Associated with each aircraft $i$, we have:

$[E_i, L_i]$: a time window during which the aircraft should ideally take-off, where $E_i$ is a release date and $L_i$ is a due date.

Also, we define:

$S_{uv}$: the minimum separation time between aircraft of weight categories $u$ and $v$ based on wake vortices.

There are additional separation constraints for aircraft flying on the same path which depend on the speed category of the leading and following aircraft.
Scheduling Aircraft Departures

A further consideration is the layout of the taxiways which may prevent overtaking and therefore some departure sequences become infeasible.

The objective function should contain a cost for violating time windows for take-off, and aim to minimize the makespan. Also, delays at holding points should be minimized.
Local Search for Aircraft Departures

Atkin, Burke, Greenwood and Reeson (2007) consider the take-off problem at Heathrow. Their model has costs associated with departure times both within and outside the time window, and delays at holding points.

They propose a tabu search algorithm in which solutions are represented as landing sequences. The neighbourhood comprises 50 randomly chosen swap and shift moves, although a random reordering of up to 5 aircraft is also allowed. An intricate procedure is used to determine feasibility of a sequence with respect to the taxiway.
Van Leeuwen, Hesselink and Rohling (2002) describe a constraint satisfaction approach for the take-off problem at Prague. Although not providing much detail about the exact model, it appears that features such as taxiways are taken into account.
Conclusions

• Various studies on aircraft landing and departure scheduling have appeared in the literature, but the models have different constraints and objective functions.

• The most widely studied solution methods are dynamic programming and local search.

• With greater availability of information in the future, the scheduling problems discussed here will change in their nature.