

EUROPEAN ORGANISATION
FOR THE SAFETY OF AIR NAVIGATION



EUROCONTROL EXPERIMENTAL CENTRE

BASE OF AIRCRAFT DATA (BADA) AIRCRAFT PERFORMANCE MODELLING REPORT

EEC Technical/Scientific Report No. 2009-009

Project: BADA

Public

Issued: March 2009

© European Organisation for the Safety of Air Navigation EUROCONTROL 2007

This document is published by EUROCONTROL in the interest of the exchange of information. It may be copied in whole or in part providing that the copyright notice and disclaimer are included. The information contained in this document may not be modified without prior written permission from EUROCONTROL.

EUROCONTROL makes no warranty, either implied or express, for the information contained in this document, neither does it assume any legal liability or responsibility for the accuracy, completeness or usefulness of this information.

REPORT DOCUMENTATION PAGE

| | | | | | | |
|---|------------------------|---|--|---------------------|---------------------------------|------------------------|
| Reference EEC Technical/Scientific Report No. 2009-009 | | Security Classification Unclassified | | | | |
| Originator: CND/VIF/ACP | | Originator (Corporate Author) Name/Location: EUROCONTROL Experimental Centre B.P.15 F – 91222 Brétigny-sur-Orge CEDEX FRANCE Telephone : +33 1 69 88 75 00 Internet : www.eurocontrol.int | | | | |
| Sponsor EUROCONTROL | | Sponsor (Contract Authority) Name/Location EUROCONTROL Agency Rue de la Fusée, 96 B –1130 BRUXELLES Telephone : +32 2 729 9011 Internet : www.eurocontrol.int | | | | |
| TITLE : BASE OF AIRCRAFT DATA (BADA) AIRCRAFT PERFORMANCE MODELLING REPORT | | | | | | |
| Author D. Poles | Date 03/2009 | Pages xii+54 | Figures 17 | Tables 14 | Annexes 2 | References 3 |
| | | Project BADA | Task no. sponsor CND/VIF/ACP | | Period 07/04 to 02/09 | |
| Distribution Statement: (a) Controlled by: Head of Section (b) Distribution : Public <input checked="" type="checkbox"/> Restricted <input type="checkbox"/> Confidential <input type="checkbox"/> (c) Copy to NTIS: YES / NO | | | | | | |
| Descriptors (keywords) : BADA, BEAM, aircraft modelling, aircraft identification, modelling process | | | | | | |
| Abstract : This document gives an overview of the aircraft performance modelling process which is used for development of the BADA family 3 aircraft models and is applicable for BADA release 3.7 onwards. Data preparation, the identification process and validation process are described as parts of the modelling process. The document introduces and describes in details the BEAM (BADA Enhanced Approach to Modelling), the new BADA family 3 model identification concept. | | | | | | |

Page intentionally left blank

FOREWORD

The scope of this document is to give an overview of the modelling process used in BADA for development of the BADA family 3 aircraft model.

Page intentionally left blank

TABLE OF CONTENTS

| | |
|--|-------------|
| FOREWORD | V |
| TABLE OF FIGURES | VIII |
| LIST OF TABLES..... | IX |
| 1. INTRODUCTION..... | 1 |
| 2. DATA PREPARATION | 3 |
| 2.1. AIRCRAFT PERFORMANCE REFERENCE DATA ACQUISITION | 3 |
| 2.1.1. Data Items Requirements..... | 3 |
| 2.1.2. Data Sources and Inherent Quality | 4 |
| 2.2. DATA PROCESSING..... | 5 |
| 2.2.1. Technical Specification Data | 6 |
| 2.2.2. Performance Data (trajectory data) | 10 |
| 2.2.3. BADA Airline Procedure Default Speed | 12 |
| 2.2.4. Buffet and Altitude Capability Files..... | 14 |
| 2.3. DATA BASE | 15 |
| 3. IDENTIFICATION PROCESS | 17 |
| 3.1. REFORMULATION OF THE BADA MODEL..... | 18 |
| 3.1.1. Engine Thrust..... | 19 |
| 3.1.2. Aerodynamic Drag..... | 23 |
| 3.1.3. Fuel Consumption | 25 |
| 3.1.4. Trajectory Generation..... | 28 |
| 3.2. THE COEFFICIENT ESTIMATION PROBLEM..... | 29 |
| 3.2.1. Multiple Trajectory Fitting | 31 |
| 3.3. MODELLING STRATEGY | 32 |
| 3.3.1. Available Data | 32 |
| 3.3.2. ISA Climb Trajectory Fitting..... | 33 |
| 3.3.3. ISA Descent Trajectory Fitting..... | 34 |
| 3.3.4. Non-ISA Trajectory Fitting | 35 |
| 3.3.5. Fuel Consumption Fitting..... | 35 |
| 3.3.6. Fitting Schema..... | 37 |
| 3.4. BEAM IDENTIFICATION TOOLBOX | 38 |
| 3.5. BEAM IDENTIFICATION PROCESS | 38 |
| 3.5.1. Identification Input Data Set | 39 |
| 3.5.2. Identification Process | 39 |
| 3.5.3. Results Analysis | 40 |
| 3.6. BEAM IMPLEMENTATION | 42 |
| 4. VALIDATION PROCESS | 43 |
| 4.1. SYNTAX VALIDATION..... | 43 |
| 4.2. CROSS VALIDATION | 43 |

4.3. RDAP VALIDATION 44

5. RESULTS.....45

6. APPENDIX A47

7. APPENDIX B51

8. REFERENCES.....55

TABLE OF FIGURES

Figure 1-1: BADA 3.7 identification process 1

Figure 2-1: Technical specification data from a model and accuracy report..... 6

Figure 2-2: Part of the list of aircraft mapped to ICAO designator P28A [2] 7

Figure 2-3: An example of performance data for a climb profile 11

Figure 2-4: An example of minimum buffet speed file..... 14

Figure 2-5: An example of altitude capability file 14

Figure 2-6: Data base import action..... 15

Figure 2-7: Data base export process..... 15

Figure 3-1: Identification process..... 17

Figure 3-2: BEAM identification process..... 39

Figure 3-3: Geopotential pressure altitude as function of time 42

Figure 3-4: The BEAM implementation diagram..... 42

Figure 4-1: RDAP overall architecture 44

Figure 6-1: Time, distance and fuel in a climb phase 47

Figure 6-2: Parameters in a climb phase 47

Figure 6-3: Time, distance, fuel in a climb phase 48

Figure 6-4: Distance, time and fuel in a descent phase..... 49

LIST OF TABLES

| | |
|--|----|
| Table 2-1: Examples of speed schedule identification | 13 |
| Table 3-1: BADA Aircraft Operational Model | 18 |
| Table 3-2: BADA model for maximum thrust | 19 |
| Table 3-3: BEAM model for maximum thrust | 20 |
| Table 3-4: BEAM model for cruise and descent thrust | 22 |
| Table 3-5: BEAM thrust coefficients as a function of thrust setting, ISA conditions and ISA type . | 22 |
| Table 3-6: Drag polar coefficient for different aerodynamic configurations in BADA | 24 |
| Table 3-7: BEAM drag coefficients as function of aerodynamic configuration and engine type | 25 |
| Table 3-8: Coefficient mapping for nominal fuel consumption for all engine types | 26 |
| Table 3-9: Coefficient mapping for minimum fuel consumption for all engine types | 27 |
| Table 3-10: BEAM fuel coefficient as a function of thrust setting and engine type | 28 |
| Table 3-11: Example of vertical speed errors | 41 |
| Table 7-1: Operations performance parameters summary table | 51 |
| Table 7-2: Airlines performance speed schedules summary table | 53 |

Page intentionally left blank

LIST OF ABBREVIATIONS

| | |
|-------------|--|
| AC | Aircraft |
| ACM5 | Aircraft Characteristics Model 5 |
| adim | adimensional |
| AOM | Aircraft Operation Manuals |
| APCH | Approach |
| APF | Airline Procedure File |
| APM | Aircraft Performance Model |
| ATM | Air Traffic Management |
| BADA | Base of Aircraft Data |
| BEAM | BADA Enhanced Approach to Modelling |
| BEW | Basic Empty Weight |
| BVP | Boundary Value Problem |
| BW | Basic Weight |
| CAS | Calibrated Airspeed |
| CMB | Climb |
| CRZ | Cruise |
| DES | Descent |
| EEC | Eurocontrol Experimental Centre |
| ESF | Energy Share Factor |
| FF | Fuel Flow |
| FL | Flight Level |
| HLDG | Holding |
| ICAO | International Civil Aviation Organization |
| ICMB | Initial Climb |
| ISA | International Standard Atmosphere |
| IVP | Initial Value Problem |
| LND | Landing |
| LS | Least Square |
| Mach | Mach number |
| MAX | maximum value |
| MEAN | mean value |
| MLW | Max Landing Weight |
| Mmo | Maximum operating Mach number |

| | |
|-------------------------|---|
| MTOW | Max Take-off Weight |
| MZFW | Max Zero Fuel Weight |
| NNLS | Non-negative Least Squares |
| ODE | Ordinary Differential Equation |
| OEW | Operating Empty Weight |
| OPF | Operation Performance File |
| PTF | Performance Table File |
| PTD | Performance Table Data File |
| RDAP | Radar Data Analysis and Processing |
| ROC | Rate of Climb |
| ROCD | Rate of Climb/Descent |
| RMS | Root-Mean-Square |
| SSE | Sum of Square Errors |
| STD | Standard Deviation |
| TAS | True Air Speed |
| TAX | Taxi |
| TEM | Total Energy Model |
| TKOF | Takeoff |
| TRJ | Trajectory |
| V_{mo} | Maximum operating speed (CAS) |
| V_{s1-g} | 1g stall speed |
| V_{smin} | Minimum stall speed |
| XML | Extensible Markup Language |

1. INTRODUCTION

The BADA modelling process, in general, may be divided in three parts:

- data related part, data preparation;
- identification process;
- validation process.

The process described in this document is related to the BADA family 3. Except for some details, the same process will be applied for development of future version of BADA family 4.

The data preparation part may be divided into three activities:

- aircraft performance reference data acquisition;
- data processing;
- data base, data manipulation and preparation for input to the identification process.

The identification process introduces a new development environment including a new identification process. The reformulation of the BADA model is described as well as a new identification tool. The BEAM (BADA Enhanced Approach to Modelling) identification process and the BEAM implementation are explained in this chapter too.

The validation process is explained at the end and it consists of:

- syntax validation;
- cross validation;
- RDAP validation.

The results are provided at the end of the modelling process and they consist of:

- documentation;
- release files.

In this document all the parts of the modelling process are described to a level of details that would allow the BADA users a better understanding of the BADA modelling process as depicted in Figure 1-1.

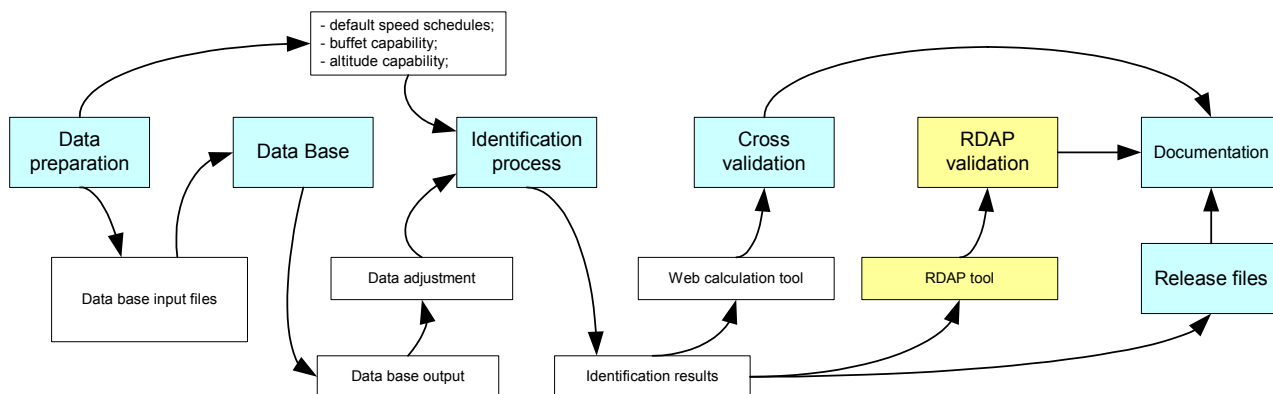


Figure 1-1: BADA 3.7 identification process

Page intentionally left blank

2. DATA PREPARATION

2.1. AIRCRAFT PERFORMANCE REFERENCE DATA ACQUISITION

Aircraft performance reference data is a key enabler for the BADA aircraft modelling process. To enable the identification of all the parameters that describe the BADA aircraft performance model and represent robustly aircraft behaviour over normal operation conditions, a variety of aircraft performance reference data items is required. The list of required data items, description of data sources and definition of data quality is provided in following chapter.

2.1.1. Data Items Requirements

Reference aircraft performance data required for the BADA modelling purposes can be grouped into the following categories:

- general characteristics;
- operating characteristics;
- aircraft performance characteristics;
- aircraft configurations and speed characteristics.

2.1.1.1. *General characteristics*

General aircraft characteristics contain the following parameters:

- type of aircraft and version identification;
- number of engines and type;
- aircraft limitation parameters:
 - Max Take-off Weight (MTOW), Max Landing Weight (MLW), Max Zero Fuel Weight (MZFW), Operating Empty Weight (OEW)/Basic weight (BW), fuel capacity, max payload, number of passengers;
 - Mean Center of Gravity position;
 - Mmo, Vmo;
 - Max operational altitude;
 - Environmental envelope;
- aircraft dimensions:
 - wing span, overall length, tail height, wing span, reference wing surface area.

2.1.1.2. *Operating characteristics*

Aircraft operating characteristics consist of the following parameters:

- typical take-off, initial climb, approach and landing configurations;
- typical speed-schedules for climb, cruise and descent, holding, alternation, turbulent weather;
- typical taxi speed on straight legs and turns

2.1.1.3. Aircraft performance characteristics

Aircraft performance characteristics consist of In-flight performances:

- flight profiles: time, distance and fuel to climb, rates-of-climb, fuel flow at climb power in function of weight, speed, altitude and different ISA conditions (ISA-20 to ISA+20); covering a range of aircraft operational speeds (low, nominal, high);
- flight profiles: time, distance and fuel to descend, rates-of-descent, fuel flow in function of weight, speed, altitude and different ISA conditions (ISA-20 to ISA+20); covering a range of operational speeds (low, nominal, high);
- fuel flow during cruise in function of weight, speed, altitude and different ISA conditions (ISA-20 to ISA+20); covering a range of operational speeds (low, nominal, high);
- speed for max endurance in function of weight, altitude, ISA conditions.

2.1.1.4. Aircraft configurations and speed characteristics

Aircraft configuration and speed characteristics are usually presented in graphs or in tables showing aircraft configurations and corresponding speeds:

- flaps/ slats/ landing gear configurations;
- stall speeds in function of weight and pressure altitude for each configuration;
- max speeds for operation at each configuration including speed brakes;
- flaps/slats retraction/extraction speed with retraction/extraction times.

2.1.2. Data Sources and Inherent Quality

The quality of the identified aircraft models directly depends on the quality of the reference data. Different aircraft manufacturers provide different reference data, with different levels of quality and quantity.

In the past, the main sources of aircraft performance reference data were Aircraft Operation Manuals (AOMs), published by aircraft manufacturers and operating airlines. From an aircraft performance modelling perspective, AOMs provide valuable information on aircraft limitations, performances and operating procedures for all aircraft types that have ever been put into operation. Aircraft performances are given in the form of integrated flight profiles that specify time, distance and fuel to climb/descent to/from specific flight level. Data is given for number of flight levels, with variable altitude steps that sometimes result in a low number of data points. Depending on the source AOM, often time to climb or decent is rounded to minute which inevitably reduces precision of the reference data and introduces significant error in the BADA model identification. Some of the examples of reference data are given in the Appendix A.

It is worth noticing that precision of the parameters provided in profile data is critical to achieve good optimization of drag, thrust and fuel flow coefficients.

Another drawback of AOMs data is that often provides aircraft profile data for only one speed schedule in climb, cruise and descent. In this case an aircraft model can be identified, but its fidelity can only be assessed and guaranteed for the range of reference data conditions.

Nowadays, aircraft manufacturers develop aircraft performance engineering programs that can provide a high quality aircraft performance reference data. These programs allow generation of reference data for a complete range of aircraft operating conditions in terms of weight, speeds, ISA and associated operating regimes with a high level of data granularity (number of data points) and data precision.

This kind of data has been recognized as an enabler for the development of better quality of aircraft performance models capable of meeting requirements for the future ATM system.

A secondary source of information is Jane's All the World's Aircraft which is published annually. Jane's is suitable for providing information such as maximum weights, dimensions, and maximum operating speeds but it does not provide reference climb or descent profiles. The data from Jane's is primarily source of information for identification of BADA synonym aircraft models.

2.2. DATA PROCESSING

As mentioned before, the aircraft performance reference data may come from different sources and be provided in different formats. In this chapter the data processing from the raw data to more structured data is described: from aircraft manufacturers' manuals, graphs and tables to the database and files that are directly used in the identification process.

There are two different groups of data that should be imported in the data base for each aircraft: technical specification data and performance data (trajectory data).

Technical specification data consists of general characteristics, operating characteristics, aircraft configurations and speed characteristics (defined in chapter 2.1).

Performance data consists of aircraft performance characteristics (again defined in 2.1).

For each aircraft an XML document containing aircraft specific and performance specific data is created. In addition, the airline performance file, buffet capability file and altitude capability file is created for each aircraft. These files are input for the identification process and are provided in pre-defined formats.

2.2.1. Technical Specification Data

Now, technical specification data will be explained in more details as well as the procedure for determining their values. Technical specification data used in the identification process are depicted in Figure 2-1. The values provided in the figure refer to one of the BADA aircraft models and have been taken as an example from an aircraft type specific modelling and accuracy report.

| Parameter | Value | Source Reference |
|---|--|--------------------|
| AC model and version identification | Airbus A321-111 | [2] |
| number of engines and engine type | 2 CFM International CFM 56-5B jet | [2] |
| ICAO wake category | M (medium) | [1] ⁽¹⁾ |
| mass (kg) | maximum: 83000 (maximum take-off) | [2] |
| | minimum: 47800 (basic operating weight) | [2] |
| | Max payload: 21700 | [2] |
| | reference: 72000 | (2) |
| maximum operating speed | CAS (knots): 350 Mach: 0.82 | [2] |
| maximum altitude (ft above sea level) | 39,000 ft | [2] |
| reference aerodynamic surface area (m ²) | 122.60 (gross wing area) | [4] ⁽²⁾ |
| stall speeds (knots, CAS) | Cruise (clean): 145 | [2] ⁽⁴⁾ |
| | Initial climb (1): 129 | |
| | Take-off (1+F): 118 | |
| | Approach (2): 106 | |
| | Landing (full): 103 | |
| climb speed | 250 kts / 300 kts / 0.78 M | [2] ⁽⁵⁾ |
| cruise speed | 250 kts / 300 kts / 0.78 M | [2] ⁽⁵⁾ |
| descent speed | 250 kts / 300 kts / 0.78 M | [2] ⁽⁵⁾ |
| BADA reference descent speed | 250 kts / 300 kts / 0.78 M | |

Figure 2-1: Technical specification data from a model and accuracy report

Aircraft model and version identification

This parameter is taken from the manufacturer’s data for each aircraft. Note that BADA modelling is done according to the ICAO designators, one BADA aircraft model for each aircraft type as distinguished by ICAO [2]. Each ICAO designator may be used for several aircraft models (and the same model with different versions). This implies that more than one aircraft model may be mapped to one BADA model. For example, on the type designator P28A 68 different aircraft models are mapped, the fragment of the list is depicted on Figure 3. At least each of the different series will have different performance (e.g. engines, airframe or other). The decision on which aircraft model will be modelled is mostly based on available aircraft performance reference data. If aircraft performance reference data are available for more than one aircraft model, the decision which aircraft model should be modelled is based on the usage in the European airspace. JP Airline Fleets International provides statistical data which is used for this decision making. It should be noted that even for one aircraft model, for the same version, it is possible to get data with different values according to the certification process (values certified according to different requests from different operating airlines). It is important to note that all further data and decisions on BADA coefficient must be consistent with the chosen aircraft model and version.



Type Designator = P28A
Total Count: 68

| Manufacturer | Model | Type Designator | Description | Engine Type | Engine Count | WTC | Photo |
|--------------|---------------------------------|-----------------|-------------|-------------|--------------|-----|-------|
| AICSA | PA-28-161 Warrior 2 | P28A | Landplane | Piston | 1 | L | - |
| AICSA | Archer 2 | P28A | Landplane | Piston | 1 | L | - |
| AICSA | Warrior 2 | P28A | Landplane | Piston | 1 | L | - |
| AICSA | PA-28-181 Cherokee Archer 2 | P28A | Landplane | Piston | 1 | L | - |
| AICSA | PA-28-181 Archer 2 | P28A | Landplane | Piston | 1 | L | - |
| AICSA | PA-28-180 Cherokee | P28A | Landplane | Piston | 1 | L | - |
| AICSA | PA-28-161 Cherokee Warrior 2 | P28A | Landplane | Piston | 1 | L | - |
| AICSA | PA-28-140 Cherokee Cruiser | P28A | Landplane | Piston | 1 | L | - |
| AICSA | PA-28-140 Cherokee | P28A | Landplane | Piston | 1 | L | - |
| AICSA | Cherokee Warrior 2 | P28A | Landplane | Piston | 1 | L | - |
| AICSA | Cherokee Cruiser | P28A | Landplane | Piston | 1 | L | - |
| AICSA | Cherokee Archer 2 | P28A | Landplane | Piston | 1 | L | - |
| AICSA | Cherokee Archer | P28A | Landplane | Piston | 1 | L | - |
| AICSA | Cherokee (PA-28-140/180) | P28A | Landplane | Piston | 1 | L | - |
| AICSA | PA-28-180 Cherokee Archer | P28A | Landplane | Piston | 1 | L | - |
| CHINCUL | Cherokee Archer | P28A | Landplane | Piston | 1 | L | - |
| CHINCUL | PA-A-28-180 Cherokee Archer | P28A | Landplane | Piston | 1 | L | - |
| CHINCUL | Warrior 2 | P28A | Landplane | Piston | 1 | L | - |
| CHINCUL | PA-A-28-181 Cherokee Archer 2 | P28A | Landplane | Piston | 1 | L | - |
| CHINCUL | PA-A-28-181 Archer 2 | P28A | Landplane | Piston | 1 | L | - |
| CHINCUL | PA-A-28-180 Cherokee Challenger | P28A | Landplane | Piston | 1 | L | - |
| CHINCUL | PA-A-28-161 Warrior 2 | P28A | Landplane | Piston | 1 | L | - |
| CHINCUL | PA-A-28-140 Cherokee Cruiser | P28A | Landplane | Piston | 1 | L | - |
| CHINCUL | Cherokee Cruiser | P28A | Landplane | Piston | 1 | L | - |
| CHINCUL | Cherokee Archer 2 | P28A | Landplane | Piston | 1 | L | - |

Figure 2-2: Part of the list of aircraft mapped to ICAO designator P28A [2]

Number of engines and engine type

The engine type and number of engines are taken from manufacturer's data. The decision which engine type will be used in modelling is based on available data following the same approach as previously explained. It is important to note that all further data and decisions must be consistent with chosen engine type.

ICAO wake category

ICAO designator and ICAO wake category are taken from the latest ICAO publications (<http://www.icao.int>) [2].

Masses

The mass values are taken from the available aircraft manufacturer's data and are specific to the aircraft model/version which is modelled in BADA. As such, the provided values in BADA may not be the same to the mass values of other aircraft model/version which belong to the same ICAO designator.

The following masses are defined in BADA:

- maximum – this value refers to maximum take-off weight (MTOW)
- minimum – this value refers to operating empty weight (OEW). In case the OEW is not available, then basic operating weight (BOW) or basic empty weight (BEW) are used instead of the OEW.
- max payload – it is defined as: $\text{Payload}_{\text{max}} = \text{MZFW} - \text{OEW}$. Depending on the data source, different ways of determining max payload are used. Sometimes the available documentation provides directly value for the maximum payload, sometimes MZFW and the OEW values are used. If none of these data items is available some estimations are done based on available information;
- reference – this is the BADA defined reference mass which is roughly 70% of the way between the minimum and maximum mass while corresponding to one of the mass values that are available in the reference trajectory data.

As mentioned above, mass values taken from manufacturer data must be consistent with the chosen aircraft model.

Maximum operating speed

Maximum operating speed in terms of CAS and Mach are taken from manufacturer's data. If not available they are taken from Jane's.

Maximum altitude

Maximum altitude is taken from manufacturer's data and if not available it is taken from Jane's. Again, this value is specific to the aircraft model/version which is modeled in BADA and may not be the same for other aircraft model/version which belong to the same ICAO designator.

Reference aerodynamic surface area

Usually, reference aerodynamic surface area is not present in manufacturer's data. If present it is taken from the manufacturer's data and if not it is taken from Jane's.

Aircraft configurations and stall speeds

Amongst the available aircraft configurations, typical configurations for five flight phases defined in BADA are selected. Then the stall speed (V_s) data at BADA reference mass are read from the manufacturer's data. Note that stall speeds in BADA are V_{smin} values, not V_{s1-g} values. If, for selected configurations, the drag-polar data are available they are used in the identification process too.

The following configuration parameters are used in the identification process :

- cruise – configuration and stall speed;
- initial climb – configuration , stall speed and drag polar data (if available);
- take-off – configuration, stall speed and drag polar data (if available);
- approach – configuration, stall speed and drag polar data (if available);
- landing – configuration, stall speed and drag polar data (if available).

Climb speed

Referent climb speed schedule is based on the recommended normal operating speeds as proposed by aircraft manufactures or operating airline. This is one of the input parameters used in the determination process of the default speed schedules. See chapter 2.2.3.

Cruise speed

Referent cruise speed schedule is based on the recommended normal operating speeds as proposed by aircraft manufactures or operating airline. This is one of the input parameters used in the determination process of the default speed schedule. See chapter 2.2.3.

Descent speed

Referent descent speed schedule is based on the recommended normal operating speeds as proposed by aircraft manufactures or operating airline. This is one of the input parameters used in the determination process of the default speed schedule. See chapter 2.2.3.

BADA reference speed schedule

BADA reference speed schedule is based on reference descent speed schedule. For the internal use only.

2.2.2. Performance Data (trajectory data)

The minimum set of trajectory data for climb, descent and cruise profiles for the BADA model generation is two (one for climb and one for descent). The recommended set of reference data for climb, descent and cruise profiles is seventeen (thirteen profiles for climb, three for descent and one for cruise). The recommended set of profiles should provide robustness of the BADA model. Details about the recommended set of profiles may be found in the chapter 3.3.1.

For all points of climb and descent profiles the position (vertical and horizontal), time and mass (fuel flow) are assumed to be known. The availability of the horizontal and vertical speed for points of climb and descent profiles is optional. If not provided in the data the values will be calculated from the available data (speed schedule and delta time). For a cruise profile only horizontal speed and instantaneous fuel consumption at each vertical position are expected to be known.

An example of performance data for a climb profile is depicted in Figure 2-3. All profile data must be inserted in a predefined XML file.

| J | K | L | M | N |
|----------------------------------|------------|------------|---------|-------|
| trajectory: lrc sp sch/mass33.64 | | A/C | | |
| delta T | 0 | deg. C | CL73 | |
| mass | 33.64 | tonnes | climb | |
| max CAS | 250 | 250 | knots | |
| max Mach | 0.700 | minimum FL | 0 | |
| LEVEL | DIST | TIME | FUEL | ROCD |
| | [n. miles] | [min] | [kg] | [fpm] |
| 450 | | | | |
| 440 | | | | |
| 430 | | | | |
| 420 | | | | |
| 410 | 283.20 | 44.70 | 1555.46 | |
| 400 | | | | |
| 390 | 144.10 | 23.90 | 1019.09 | |
| 380 | | | | |
| 370 | 115.10 | 19.60 | 894.09 | |
| 360 | | | | |
| 350 | 98.80 | 17.20 | 817.27 | |
| 340 | | | | |
| 330 | 87.10 | 15.40 | 757.73 | |
| 320 | | | | |
| 310 | 75.50 | 13.70 | 694.55 | |
| 300 | | | | |
| 290 | 64.90 | 12.10 | 632.27 | |
| 280 | | | | |
| 270 | 56.10 | 10.70 | 575.91 | |
| 260 | | | | |
| 250 | 48.60 | 9.50 | 523.64 | |
| 240 | | | | |
| 230 | | | | |
| 220 | | | | |
| 210 | | | | |
| 200 | 33.60 | 6.90 | 405.45 | |
| 190 | | | | |
| 180 | | | | |
| 170 | | | | |
| 160 | | | | |
| 150 | 22.40 | 4.80 | 298.64 | |
| 140 | | | | |
| 130 | | | | |
| 120 | | | | |
| 110 | | | | |
| 100 | 13.60 | 3.10 | 197.73 | |
| 90 | | | | |
| 80 | | | | |
| 70 | | | | |
| 60 | | | | |
| 50 | 6.30 | 1.50 | 98.64 | |
| 40 | | | | |
| 30 | | | | |
| 20 | | | | |
| 15 | 1.8 | 0.4 | 30 | |
| 10 | | | | |
| 0 | 0 | 0 | 0 | |

Figure 2-3: An example of performance data for a climb profile

2.2.3. BADA Airline Procedure Default Speed

For each BADA aircraft model, so called airline procedure default speed is provided for climb, cruise and descent phases. For each phase of flight three speed values are provided:

V_1 - standard CAS (knots) below 10,000 ft;

V_2 - standard CAS (knots) between 10,000 ft and Mach transition altitude;

M - standard Mach number above Mach transition altitude;

These values are determined taking into account information on aircraft operations coming from:

- aircraft manufacturers' documentation which provides typical or nominal operating speeds for each flight phase;
- operational data: radar recordings and flight plans from several locations, covering many aircraft types and airlines;
- pilots' and Air Traffic Controllers' input based on their 'real life' experiences.

The following procedure is used to determine the default values for each BADA model, flight phase and speed type:

1. From the set of trajectories and performance data used to identify the BADA model, the range of speed values for which the model has been identified is determined.
2. From the available operational data, the mean observed speed value is determined.
3. From the values determined in 1) and 2), the speed observed in real flights is compared to the speed range used for the modelling, and an educated guess is made to select the proposed speed value. Table 1 presents several examples of the rules used to determine a speed value that is both close to the observed value and in agreement with the modelling data.

Table 2-1: Examples of speed schedule identification

| ICAO identifier | Speed kind | Min. value used for modelling | Max. value used for modelling | Value observed in recordings | Proposed default value | Comments |
|-----------------|-----------------|-------------------------------|-------------------------------|------------------------------|------------------------|---|
| B736 | Climb CAS | 260 | 300 | 290 | 290 | <i>The value identified from the recordings fits in the modelling envelope: This observed value is proposed</i> |
| T134 | Climb CAS | 270 | 270 | - | 270 | <i>No recordings available and only one value used for modelling: This modelling value is proposed</i> |
| CL60 | Descent Mach | 0.78 | 0.78 | 0.82 | 0.82 | <i>The value identified from the recordings is outside of the modelling envelope, but this envelope was limited to only one value: The observed value is proposed</i> |

The speed schedules obtained in this procedure are used as speed schedules in airline procedure files (APF) for each BADA model.

2.2.4. Buffet and Altitude Capability Files

Information on altitude capability and minimum buffet speeds (valid only for jet aircraft models) are taken from manufacturer’s data. Examples of altitude capability and buffet data are shown on Figure 2-4 and Figure 2-5.

| | A | B | C |
|---|----------------|----------|-------|
| 1 | Aircraft type: | A/C-type | |
| 2 | N = | 1.2 | |
| 3 | FL | WEIGHT | Mach |
| 4 | 350 | 272727 | 0.745 |
| 5 | 300 | 272727 | 0.67 |
| 6 | 370 | 204545 | 0.68 |
| 7 | 270 | 204545 | 0.52 |

Figure 2-4: An example of minimum buffet speed file

| | A | B | C |
|----|-----------------------|--------------|-------------|
| 1 | Aircraft type: | A/C-type | |
| 2 | Speed law: | 310/310/0.78 | |
| 3 | MTOW (kgs): | 77000 | |
| 4 | Hmo (ft): | 39000 | |
| 5 | R/C (ft/min): | 300 | |
| 6 | | | |
| 7 | Weight | ISA | Hmax |
| 8 | 60674 | 0 | 39000 |
| 9 | 64000 | 0 | 37962 |
| 10 | 70492 | 0 | 36089 |
| 11 | 77000 | 0 | 34354 |
| 12 | 60421 | 10 | 39000 |
| 13 | 64000 | 10 | 37886 |
| 14 | 70196 | 10 | 36089 |
| 15 | 77000 | 10 | 34269 |
| 16 | 52270 | 20 | 39000 |
| 17 | 64000 | 20 | 36942 |
| 18 | 67002 | 20 | 36089 |
| 19 | 77000 | 20 | 32689 |

Figure 2-5: An example of altitude capability file

Buffet and altitude capability files are used directly in the identification process (see Figure 1-1).

2.3. DATA BASE

An Oracle based data base is used as the BADA aircraft data repository. Here, the import and export actions are explained while the data organization and relations are not within the scope of the document. For the data exchange purposes the web based interface is used. In the web interface the action and input files may be chosen. In addition to the data, a file (paramFM XML file) that describes the import action must be created and used for the import action. The import action is shown on the Figure 2-6.

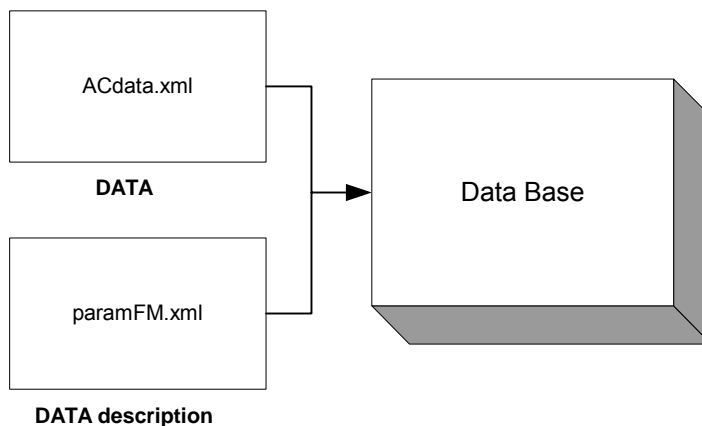


Figure 2-6: Data base import action

In order to export data the aircraft type should be selected and the name for output files should be inserted in the web interface. In addition, data filter may be used to get more precise data. Five output files are provided: request.txt that describes the request and four files that are related to the aircraft. Files that describe the aircraft are given in XML format and include: trajectory data (name_data.xml), operation performance data (name_OPF_id.xml), summary of exported trajectory data (name_resume.xml) and used filter parameters in exporting process (name_filter.xml). Note that just name_data.xml and name_OPF_id.xml files are used in the identification process. In addition, note that the output OPF XML file and an identified OPF file are not the same but most of parameters are the same (more precisely non-identified parameters are the same). The exporting process is given in Figure 2-7.

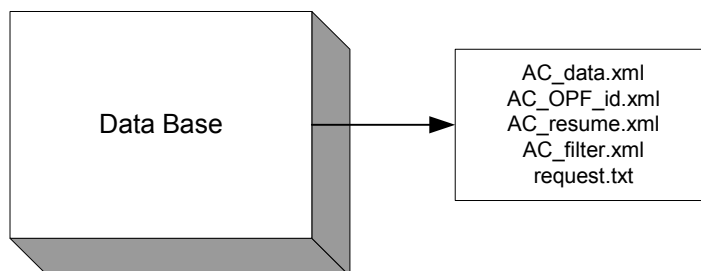


Figure 2-7: Data base export process

Page intentionally left blank

3. IDENTIFICATION PROCESS

For the BADA release 3.7 a new development environment including the new identification process were used. The new concept is called BEAM and it is based on Matlab.

The problem of estimating BADA coefficients is identified as a non-linear coupled multivariate parameter estimation problem. However, the new formulation introduced allows showing that the non-linearity and couple-ness characteristics are small and can be avoided by means of an iterative scheme. This scheme splits the problem into several linear sub-problems whose sub-optimal solutions progressively incorporate more reference data and estimates coming from other sub-problems until the global optimal solution is achieved. Instead of dealing with each engine type, aerodynamic configuration and thrust level setting as separate models, generalized models are introduced for aerodynamic drag, engine thrust and fuel consumption, that are valid for all cases. The solution heavily relies on the well-founded and powerful technique of least squares linear fitting. The objective is to minimize the sum of square errors in the derivatives of the ROC and fuel consumption, whose integration provides the BADA trajectories, thus, obtaining the best fit in the sense of RMS (root-mean-square). The identification process is depicted on Figure 3-1.

In this chapter the reformulation of the BADA model and new identification tool, together with the BEAM identification process and the BEAM implementation are described.

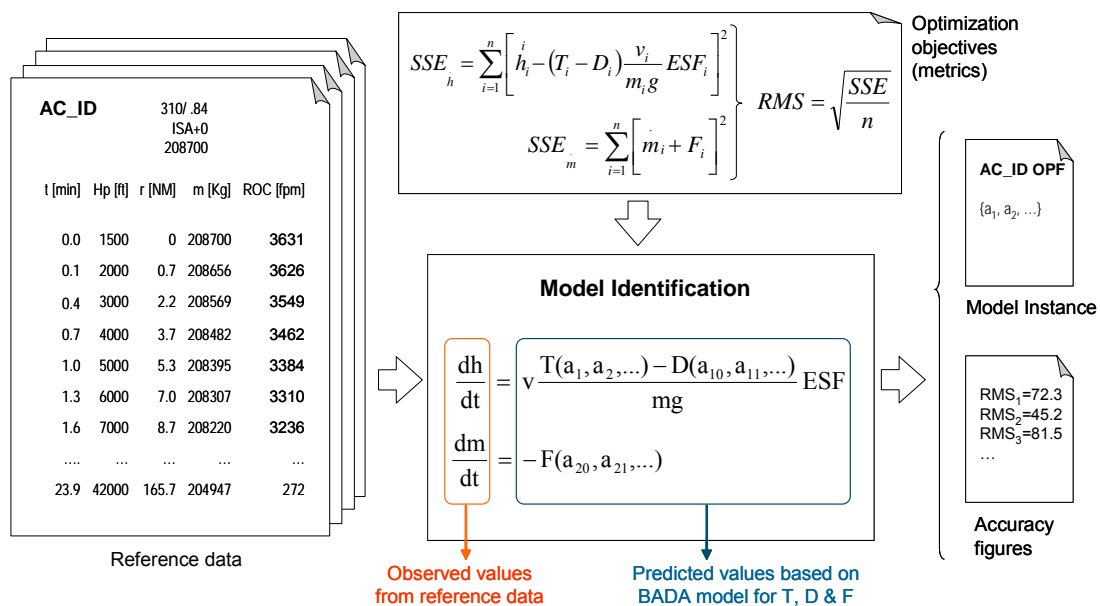


Figure 3-1: Identification process

At the end of the identification process it is assumed that the BADA model, performance table file (PTF) and performance table data file (PTD) for each aircraft are available. Under the BADA model for an aircraft, the parameters provided in operation performance file (OPF file) and airline performance file (APF file) are considered, i.e. the BADA model for an aircraft consists of OPF and APF files for that particular aircraft. The BADA models are results of the BEAM identification process while the PTF and PTD files are results of the BEAM implementation.

3.1. REFORMULATION OF THE BADA MODEL

The BADA *aircraft operation model* used is shown in Table 3-1. Although not considered in [1], *holding* and *ground* stages have been added for the sake of completeness.

Table 3-1: BADA Aircraft Operational Model

| Stage | Phase | Aerodynamic configuration | Thrust setting |
|---------|-------|---------------------------|----------------|
| Climb | TKOF | tkof | max_cmb |
| | ICMB | icmb | |
| | CMB | clean | |
| Cruise | CRZ | | max_crz |
| Holding | HLDG | | des_hi |
| Descent | DES | | |
| | APCH | apch | des_apch |
| | LDG | ldg | des_ldg |
| Ground | TAX | clean | idle |

3.1.1. Engine Thrust

Maximum thrust under ISA conditions

Current BADA model for the calculation of the maximum thrust under ISA conditions for the three different engine types [1] is shown in the Table 3-2.

Table 3-2: BADA model for maximum thrust

| Engine type | BADA expression for $(T_{\max \text{ climb}})_{\text{ISA}}$ [N] | units |
|-------------|--|--|
| Jet | $(T_{\max \text{ climb}})_{\text{ISA}} = C_{\text{Te1}} \left(1 - \frac{h}{C_{\text{Te2}}} + C_{\text{Te3}} h^2 \right)$ | $[C_{\text{Te1}}]=\text{N}$ $[C_{\text{Te2}}]=\text{ft}$ $[C_{\text{Te3}}]=1/\text{ft}^2$ |
| Turboprop | $(T_{\max \text{ climb}})_{\text{ISA}} = \frac{C_{\text{Te1}}}{V_{\text{TAS}}} \left(1 - \frac{h}{C_{\text{Te2}}} \right) + C_{\text{Te3}}$ | $[C_{\text{Te1}}]=\text{kt}\cdot\text{N}$ $[C_{\text{Te2}}]=\text{ft}$ $[C_{\text{Te3}}]=\text{N}$ |
| Piston | $(T_{\max \text{ climb}})_{\text{ISA}} = C_{\text{Te1}} \left(1 - \frac{h}{C_{\text{Te2}}} \right) + \frac{C_{\text{Te3}}}{V_{\text{TAS}}}$ | $[C_{\text{Te1}}]=\text{N}$ $[C_{\text{Te2}}]=\text{ft}$ $[C_{\text{Te3}}]=\text{kt}\cdot\text{N}$ |

For jet aircraft, the expression given in Table 3-2 can be rearranged so that the maximum thrust depends on the coefficients in a linear way:

$$(T_{\max \text{ climb}})_{\text{ISA}} = C_{\text{Te1}} - \frac{C_{\text{Te1}}}{C_{\text{Te2}}} h + C_{\text{Te1}} C_{\text{Te3}} h^2 \tag{3.1-1}$$

In effect, if the new coefficients t_0 , t_1 and t_4 are introduced, the above expression can be rewritten as:

$$(T_{\max \text{ climb}})_{\text{ISA}} = t_0 - t_1 h + t_4 h^2 \tag{3.1-2}$$

with $[t_0]=\text{N}$, $[t_1]=\text{N}/\text{ft}$ and $[t_4]=\text{N}/\text{ft}^2$, and:

$$t_0 = C_{\text{Te1}} \quad t_1 = \frac{C_{\text{Te1}}}{C_{\text{Te2}}} \quad t_4 = C_{\text{Te1}} C_{\text{Te3}}$$

or: $C_{\text{Te1}} = t_0 \quad C_{\text{Te2}} = \frac{t_0}{t_1} \quad C_{\text{Te3}} = \frac{t_4}{t_0}$

Analogous arrangements can be made for the other engine types. For turboprop,

$$(T_{\max \text{ climb}})_{\text{ISA}} = t_0 + t_2 \frac{1}{V_{\text{TAS}}} - t_3 \frac{h}{V_{\text{TAS}}} \tag{3.1-3}$$

with $[t_0]=N$, $[t_2]=N \cdot kt$ and $[t_3]=N \cdot kt/ft$, and:

$$t_0 = C_{Tc3} \quad t_2 = C_{Tc1} \quad t_3 = \frac{C_{Tc1}}{C_{Tc2}}$$

or: $C_{Tc1} = t_2 \quad C_{Tc2} = \frac{t_2}{t_3} \quad C_{Tc3} = t_0$

And for piston:

$$(\Gamma_{\max \text{ climb}})_{ISA} = t_0 - t_1 h + t_2 \frac{1}{V_{TAS}} \tag{3.1-4}$$

with $[t_0]=N$, $[t_1]=N/ft$ and $[t_2]=N \cdot kt$, and:

$$t_0 = C_{Tc1} \quad t_1 = \frac{C_{Tc1}}{C_{Tc2}} \quad t_2 = C_{Tc3}$$

or: $C_{Tc1} = t_0 \quad C_{Tc2} = \frac{t_0}{t_1} \quad C_{Tc3} = t_2$

Table 3-3 summarizes the above BEAM expressions for $(\Gamma_{\max \text{ climb}})_{ISA}$.

Table 3-3: BEAM model for maximum thrust

| Engine type | BEAM expression for $(\Gamma_{\max \text{ climb}})_{ISA}$ [N] | Units |
|-------------|---|--|
| Jet | $(\Gamma_{\max \text{ climb}})_{ISA} = t_0 - t_1 h + t_4 h^2$ | [h]=ft |
| Turboprop | $(\Gamma_{\max \text{ climb}})_{ISA} = t_0 + t_2 \frac{1}{V_{TAS}} - t_3 \frac{h}{V_{TAS}}$ | [V _{TAS}]=kt [t ₀]=N |
| Piston | $(\Gamma_{\max \text{ climb}})_{ISA} = t_0 - t_1 h + t_2 \frac{1}{V_{TAS}}$ | [t ₁]=N/ft [t ₂]=N·kt [t ₃]=N·kt/ft [t ₄]=N/ft ² |

With the help of the introduced BEAM coefficients, $(\Gamma_{\max \text{ climb}})_{ISA}$ can be generalized as:

$$(\Gamma_{\max \text{ climb}})_{ISA} = t_0 - t_1 h + t_2 \frac{1}{V_{TAS}} - t_3 \frac{h}{V_{TAS}} + t_4 h^2 \tag{3.1-5}$$

which is a valid expression for all the three: jet ($t_2=t_3=0$), turboprop ($t_1=t_4=0$) and piston ($t_3=t_4=0$) aircraft, with $t_0, \dots, t_4 \geq 0$ to be applied as a constraint.

Thrust correction for temperature deviation from ISA

In the current BADA specification [1] the correction for temperature deviation ΔT_{ISA} above ISA standard conditions is modelled as:

$$T_{\max \text{ climb}} = (T_{\max \text{ climb}})_{ISA} [1 - C_{Tc5}(\Delta T_{ISA} - C_{Tc4})] \quad (3.1-6)$$

with the constraints:

$$\text{i) } 0 \leq (\Delta T_{ISA} - C_{Tc4})C_{Tc5} \leq 0.4$$

$$\text{ii) } C_{Tc5} \geq 0$$

Again, the above expression can be developed:

$$T_{\max \text{ climb}} = (T_{\max \text{ climb}})_{ISA} (1 - C_{Tc5} \Delta T_{ISA} + C_{Tc5} C_{Tc4}) \quad (3.1-7)$$

and rearranged as:

$$T_{\max \text{ climb}} = (T_{\max \text{ climb}})_{ISA} (t_6 - t_5 \Delta T_{ISA}) \quad (3.1-8)$$

were:

$$t_5 = C_{Tc5} \quad t_6 = 1 + C_{Tc4} C_{Tc5}$$

$$\text{or: } C_{Tc4} = \frac{t_6 - 1}{t_5} \quad C_{Tc5} = -t_5$$

with $[t_5]=1/K$ and t_6 is dimensionless. The expression shows that the correction for temperature deviation from ISA standard conditions is linear with respect to the BEAM coefficients t_5 and t_6 .

The constraints expressed in terms of the new BEAM coefficients are:

$$\text{i) } 0 \leq 1 - t_6 - t_5 \Delta T_{ISA} \leq 0.4 \quad \Rightarrow \quad 0.6 + t_5 \Delta T_{ISA} \leq t_6 \leq 1 + t_5 \Delta T_{ISA}$$

$$\text{ii) } t_5 \geq 0$$

Maximum cruise thrust and descent thrust

Table 3-4 summarizes the expressions of the maximum cruise thrust and descent thrust for various descent configurations [1]. For all these configurations the corresponding thrust can be modelled as:

$$T = t_7 T_{\max \text{ climb}} \quad (3.1-9)$$

t_7 being the appropriate thrust coefficient depending on the thrust setting. The above formula is also valid for maximum thrust by making $t_7=1$.

Table 3-4: BEAM model for cruise and descent thrust

| Engine type | Thrust setting | BADA expression for thrust [N] | Units |
|------------------------------|----------------|--|---|
| jet, turboprop, piston | max_crz | $T_{\text{max cruise}} = C_{Tcr} T_{\text{max climb}}$ | $[T_{\text{max climb}}]=\text{N}$ |
| | des_hi | $T_{\text{des,high}} = C_{Tdes,high} T_{\text{max climb}}$ | $[C_{Tcr}]=\text{adim}$ |
| | des_lo | $T_{\text{des,low}} = C_{Tdes,low} T_{\text{max climb}}$ | $[C_{Tdes,high}]=\text{adim}$ |
| | des_app | $T_{\text{des,app}} = C_{Tdes,app} T_{\text{max climb}}$ | $[C_{Tdes,low}]=\text{adim}$ |
| | des_ldg | $T_{\text{des,ld}} = C_{Tdes,ld} T_{\text{max climb}}$ | $[C_{Tdes,app}]=\text{adim}$ $[C_{Tdes,ld}]=\text{adim}$ |

Generalized BEAM thrust model

A generalized thrust model (equivalent to the BADA model but more appropriate for the coefficient estimation purposes) is introduced within the BEAM context that is applicable to all engine types, temperature conditions and thrust level settings. The following expression defines this model:

$$T = t_7 \left(t_0 - t_1 h + t_2 \frac{1}{V_{TAS}} - t_3 \frac{h}{V_{TAS}} + t_4 h^2 \right) (t_6 - t_5 \Delta T_{ISA}) \tag{3.1-10}$$

This generalized thrust model should be applied according to the schema shown in Table 3-5.

Table 3-5: BEAM thrust coefficients as a function of thrust setting, ISA conditions and ISA type

| Engine type | Thrust setting | | | | | |
|------------------------------|---|-----------------|---------------------------------|----------------------|---------------------------------|---------------------|
| | max_cmb | max_crz | des_hi | des_lo | des_apch | des_ldg |
| Jet | | $t_0 = C_{Tc1}$ | $t_1 = \frac{C_{Tc1}}{C_{Tc2}}$ | $t_2 = t_3 = 0$ | $t_4 = C_{Tc1} C_{Tc3}$ | |
| Turboprop | | $t_0 = C_{Tc3}$ | $t_1 = 0$ | $t_2 = C_{Tc1}$ | $t_3 = \frac{C_{Tc1}}{C_{Tc2}}$ | $t_4 = 0$ |
| Piston | | $t_0 = C_{Tc1}$ | $t_1 = \frac{C_{Tc1}}{C_{Tc2}}$ | $t_2 = C_{Tc3}$ | $t_3 = t_4 = 0$ | |
| Jet, turboprop, piston | $t_5 = t_6 = 0 \quad (\Delta T_{ISA} = 0)$ | | | | | |
| | $t_5 = C_{Tc5} \quad t_6 = 1 + C_{Tc4} C_{Tc5} \quad (\Delta T_{ISA} \neq 0)$ | | | | | |
| | $t_7 = 1$ | $t_7 = C_{Tcr}$ | $t_7 = C_{Tdes,high}$ | $t_7 = C_{Tdes,low}$ | $t_7 = C_{Tdes,app}$ | $t_7 = C_{Tdes,ld}$ |

As shown in Table 3-5, the so-called BEAM thrust coefficients $\{t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7\}$ are bi-univocally related to BADA thrust coefficients $\{C_{Tc1}, C_{Tc2}, C_{Tc3}, C_{Tc4}, C_{Tc5}, C_{Tcr}, C_{Tdes,high}, C_{Tdes,low}, C_{Tdes,app}, C_{Tdes,id}\}$ once the engine type, thrust setting and ISA temperature conditions are known. In addition, the generalized BEAM thrust model has been defined so that coefficients t_0, \dots, t_4 are all non-negative.

3.1.2. Aerodynamic Drag

In [1], drag force, D [N], is given by the expression:

$$D = \frac{1}{2} \rho k_1 V_{TAS}^2 S C_D, \quad (3.1-11)$$

$$\text{with } k_1 = \left(\frac{1852}{3600} \right)^2 \left[\frac{(m/s)^2}{kt^2} \right]$$

where the drag coefficient C_D [adim] is a function of the lift coefficient C_L [adim] and Mach number M [adim], known as *drag polar*. Air density ρ is in $[kg/m^3]$, true airspeed V_{TAS} is in $[kt]$ and wing reference area S in $[m^2]$.

Drag polar

In the current BADA specification [1], the drag polar is modelled as:

$$C_D = C_{D0} + C_{D2} C_L^2 \quad (3.1-12)$$

The lift coefficient C_L is determined taking into account the path angle γ and the bank angle ϕ as:

$$C_L = \frac{2mg \cos \gamma}{\rho k_1 V_{TAS}^2 S \cos \phi} \quad (3.1-13)$$

where m is the aircraft mass $[kg]$, and γ is set to zero for all aircraft models. Substituting (3.1-12) into (3.1-11) and (3.1-11) yields:

$$D = \frac{1}{2} \rho k_1 V_{TAS}^2 S C_{D0} + \frac{2m^2 g^2 \cos^2 \gamma}{\rho k_1 V_{TAS}^2 S \cos^2 \phi} C_{D2} \quad (3.1-14)$$

If the following coefficients are introduced,

$$d_0 = \frac{1}{2} k_1 S C_{D0} \quad d_2 = \frac{2g^2}{k_1 S} C_{D2}$$

the model for aerodynamic drag results in:

$$D = d_0 \rho V_{TAS}^2 + d_2 \frac{m^2 \cos^2 \gamma}{\rho V_{TAS}^2 \cos^2 \phi} \quad (3.1-15)$$

Table 3-6: Drag polar coefficient for different aerodynamic configurations in BADA

| Engine type | Aerodynamic configuration | C_{D0} | C_{D2} |
|------------------------------|---------------------------|---------------------------------|--------------|
| Jet, turboprop, piston | tkof | $C_{D0,TO}$ | $C_{D2,TO}$ |
| | icmb | $C_{D0,IC}$ | $C_{D2,IC}$ |
| | clean | $C_{D0,CR}$ | $C_{D2,CR}$ |
| | apch | $C_{D0,APP}$ | $C_{D2,APP}$ |
| | ldg | $C_{D0,LD} + C_{D0,\Delta LDG}$ | $C_{D2,LD}$ |

In [1] the drag polar coefficient C_{D0} and C_{D2} have different values depending on the aerodynamic configuration, as shown in Table 3-6.

Drag correction for compressibility effect

At high Mach numbers (namely $M > 0.7$) compressibility effects take place, making the drag increase significantly more than the parabolic drag polar model used in [1] predicts. To account for these effects the following model is proposed in BADA (although it is not currently implemented).

$$C_D = (C_{D0} + C_{D2} C_L^2)(1 + C_{M16} M^{16}) \quad (3.1-16)$$

where $(1 + C_{M16} M^{16})$ is the term that accounts for compressibility effects. (C_{M16} is dimensionless and set to zero for aircraft models in the current BADA identification).

Introducing (3.1-16) instead of (3.1-12) in (3.1-11) yields the drag model corrected for compressibility effects:

$$D = \left(d_0 \rho V_{TAS}^2 + d_2 \frac{m^2 \cos^2 \gamma}{\rho V_{TAS}^2 \cos^2 \phi} \right) (1 + C_{M16} M^{16}) \quad (3.1-17)$$

The model described by this expression (3.1-17) is only intended for a clean configuration, the only one in which high Mach numbers can be achieved.

Generalized BEAM drag model

A generalized drag model (equivalent to the BADA model but more appropriate for coefficient estimation purposes) is introduced within the BEAM context applicable for all aerodynamic configurations and flight conditions. The following expression defines this model:

$$D = \left(d_0 \rho V_{TAS}^2 + d_2 \frac{m^2 \cos^2 \gamma}{\rho V_{TAS}^2 \cos^2 \phi} \right) (1 + d_{16} M^{16}) \quad (3.1-18)$$

with the BEAM drag coefficients provided by the application schema showed in Table 3-7.

As shown in Table 3-7, the so-called BEAM drag coefficients $\{d_0, d_2, d_{16}\}$ are bi-univocally related to BADA drag coefficients $\{C_{D0,TO}, C_{D2,TO}, C_{D0,IC}, C_{D2,IC}, C_{D0,CR}, C_{D2,CR}, C_{M16}, C_{D0,APP}, C_{D2,APP}, C_{D0,LD}, C_{D0,\Delta LDG}, C_{D2,LD}\}$ once the engine type and aerodynamic configuration are known. In addition, the generalized BEAM drag model has been defined so that all drag coefficients are non-negative.

Table 3-7: BEAM drag coefficients as function of aerodynamic configuration and engine type

| Aerodynamic configuration | Engine type | | |
|---------------------------|--|-------------------------------------|--------------------|
| | Jet | Turboprop | Piston |
| tkof | $d_0 = \frac{1}{2}k_1SC_{D0,TO}$ | $d_2 = \frac{2g^2}{k_1S}C_{D2,TO}$ | $d_{16} = 0$ |
| icmb | $d_0 = \frac{1}{2}k_1SC_{D0,IC}$ | $d_2 = \frac{2g^2}{k_1S}C_{D2,IC}$ | $d_{16} = 0$ |
| clean | $d_0 = \frac{1}{2}k_1SC_{D0,CR}$ | $d_2 = \frac{2g^2}{k_1S}C_{D2,CR}$ | $d_{16} = C_{M16}$ |
| apch | $d_0 = \frac{1}{2}k_1SC_{D0,APP}$ | $d_2 = \frac{2g^2}{k_1S}C_{D2,APP}$ | $d_{16} = 0$ |
| ldg | $d_0 = \frac{1}{2}k_1S(C_{D0,LD} + C_{D0,\Delta LDG})$ | $d_2 = \frac{2g^2}{k_1S}C_{D2,LD}$ | $d_{16} = 0$ |

3.1.3. Fuel Consumption

Nominal fuel consumption

For jet and turboprop aircraft, nominal fuel consumption, f_{nom} [kg/s] is obtained as the product of the thrust T [N] and the thrust specific fuel consumption η [kg/(min·kN)]:

$$f_{nom} = k_2 \eta T, \quad (3.1-19)$$

$$\text{with } k_2 = \frac{1}{60 \cdot 10^3} \left[\frac{\text{kN} \cdot \text{min}}{\text{N} \cdot \text{s}} \right]$$

η is modelled as a function of the true airspeed V_{TAS} [kt].

For jet:

$$\eta = C_{f1} \left(1 + \frac{V_{TAS}}{C_{f2}} \right) \quad (3.1-20)$$

with $[C_{f1}] = \text{Kg}/(\text{min} \cdot \text{kN})$ and $[C_{f2}] = \text{kt}$, whereas for turboprop:

$$\eta = C_{f1} \left(1 - \frac{V_{TAS}}{C_{f2}} \right) \frac{V_{TAS}}{10^3} \quad (3.1-21)$$

with $[C_{f1}] = \text{Kg}/(\text{min} \cdot \text{kN} \cdot \text{kt})$ and $[C_{f2}] = \text{kt}$.

For piston engines, the nominal fuel consumption f_{nom} [kg/s] is modelled so that it does not depend on the thrust:

$$f_{nom} = k_3 C_{f1} \quad (3.1-22)$$

where $k_3 = \frac{1}{60}$ [min/s]

and $[C_{f1}] = \text{kg}/\text{min}$.

Therefore the following model is proposed for f_{nom} as to generalize the above expressions:

$$f_{nom} = (f_2 + f_3 V_{TAS} - f_4 V_{TAS}^2) T + f_0 \quad (3.1-23)$$

with $[f_0] = \text{kg}/\text{s}$, $[f_2] = \text{kg}/\text{s}$, $[f_3] = \text{Kg}/(\text{s} \cdot \text{kt})$ and $[f_4] = \text{kg}/(\text{s} \cdot \text{kt}^2)$.

According to the expressions above, the relationships between the BADA coefficients and the new ones introduced can be summarized as showed in Table 3-8.

Table 3-8: Coefficient mapping for nominal fuel consumption for all engine types

| Engine type | BEAM coefficients | | | | BADA coefficients | |
|-------------|--------------------|--------------------|-----------------------------------|--|---------------------------------|----------------------------|
| Jet | $f_0 = 0$ | $f_2 = k_2 C_{f1}$ | $f_3 = k_2 \frac{C_{f1}}{C_{f2}}$ | $f_4 = 0$ | $C_{f1} = \frac{f_2}{k_2}$ | $C_{f2} = \frac{f_2}{f_3}$ |
| Turboprop | $f_0 = 0$ | $f_2 = 0$ | $f_3 = \frac{k_2}{10^3} C_{f1}$ | $f_4 = \frac{k_2}{10^3} \frac{C_{f1}}{C_{f2}}$ | $C_{f1} = \frac{10^3}{k_2} f_3$ | $C_{f2} = \frac{f_3}{f_4}$ |
| Piston | $f_0 = k_3 C_{f1}$ | $f_2 = 0$ | $f_3 = 0$ | $f_4 = 0$ | $C_{f1} = \frac{f_0}{k_3}$ | $C_{f2} = 0$ |

Minimum fuel consumption

According to [1], for idle thrust or descent conditions, the corresponding fuel consumption is the minimum fuel consumption f_{min} [kg/s]. For jet and turboprop aircraft, f_{min} is modelled as a function of the altitude h [ft]:

$$f_{\min} = k_3 C_{f3} \left(1 - \frac{h}{C_{f4}} \right) \quad (3.1-24)$$

with $[C_{f3}] = \text{kg}/\text{min}$ and $[C_{f4}] = \text{ft} \cdot \text{min}/\text{kg}$, while for piston aircraft, f_{\min} is a constant:

$$f_{\min} = k_3 C_{f3} \quad (3.1-25)$$

Again, generalization for f_{\min} is proposed:

$$f_{\min} = f_0 - f_1 h \quad (3.1-26)$$

with $[f_1] = \text{kg}/(\text{s} \cdot \text{ft})$.

And the corresponding relationships between BADA and BEAM coefficients are summarized in Table 3-9.

Table 3-9: Coefficient mapping for minimum fuel consumption for all engine types

| Engine type | BEAM coefficients | | BADA coefficients | |
|-------------------|--------------------|-----------------------------------|----------------------------|----------------------------|
| jet, turboprop | $f_0 = k_3 C_{f3}$ | $f_1 = k_3 \frac{C_{f3}}{C_{f4}}$ | $C_{f3} = \frac{f_0}{k_3}$ | $C_{f4} = \frac{f_0}{f_1}$ |
| piston | | $f_1 = 0$ | | $C_{f4} = 0$ |

Fuel consumption for cruise

In current BADA specification [1], fuel consumption for the cruise phase is modelled as a fraction of the nominal fuel consumption:

$$f_{\text{cr}} = C_{\text{fcr}} f_{\text{nom}} \quad [C_{\text{fcr}}] = \text{adim} \quad (3.1-27)$$

The above expression is valid for all the three engine types. For the sake of generality, the coefficient f_5 [adim] is introduced:

$$f_5 = C_{\text{fcr}} \quad \text{for cruise} \quad (3.1-28)$$

$$f_5 = 1 \quad \text{for all other conditions.} \quad (3.1-29)$$

Generalized BEAM fuel consumption model

From the prior expressions, a model (equivalent to the BADA model but more suitable for coefficient estimation purposes) is introduced within the BEAM context, that generalizes the fuel consumption for all engine types, phases of the flight and thrust level settings. Following expression defines this model:

$$F = f_5 [f_0 - f_1 h + (f_2 + f_3 V_{\text{TAS}} - f_4 V_{\text{TAS}}^2) T], \quad (3.1-30)$$

with [F]=kg/s.

The generalized fuel consumption model should be applied according to the schema of the Table 3-10.

As shown in Table 3-9, the so-called BEAM fuel coefficients $\{f_0, f_1, f_2, f_3, f_4, f_5\}$ are bi-univocally related with BADA drag coefficients $\{C_{f1}, C_{f2}, C_{f3}, C_{f4}, C_{fcr}\}$ once the engine type and thrust setting are known. In addition, the generalized BEAM fuel model has been defined so that all fuel coefficients are non-negative.

Table 3-10: BEAM fuel coefficient as a function of thrust setting and engine type

| Engine type | Thrust setting | | | | | | |
|-------------|----------------|--|-----------------|---|-------------------------------|--------------------|-----------------------------------|
| | max_cmb | max_crz | | des_hi, des_lo, des_apch, des_ldg, idle | | | |
| Jet | $f_5=1$ | $f_0=f_1=f_4=0$ $f_2 = k_2 C_{f1}$ $f_3 = k_2 \frac{C_{f1}}{C_{f2}}$ | $f_5 = C_{fcr}$ | $f_5=1$ | $f_2=0$ $f_3=0$ $f_4=0$ | $f_0 = k_3 C_{f3}$ | $f_1 = k_3 \frac{C_{f3}}{C_{f4}}$ |
| Turboprop | | $f_0=f_1=f_2=0$ $f_3 = \frac{k_2}{10^3} C_{f1}$ $f_4 = \frac{k_2}{10^3} \frac{C_{f1}}{C_{f2}}$ | | | | | |
| Piston | | $f_1=f_3=f_4=0$ $f_0 = k_3 C_{f1}$ | | | | | $f_1=0$ |

3.1.4. Trajectory Generation

The BADA total energy model (TEM) in the form that appears in [1] (3.1-4), together with the mass variation and horizontal distance variation laws, form the following ordinary first order differential equations (ODE) system:

$$\frac{dh}{dt} = k_4 (T - D) \frac{V_{TAS}}{mg} ESF, \tag{3.1-31}$$

with $k_4 = \frac{1852}{0.3048 \times 60}$ [fpm/kt]

$$\frac{dr}{dt} = k_3 V_{TAS} \cos \gamma, \quad (3.1-32)$$

with $k_3 = \frac{1}{60}$ [hour/min]

$$\frac{dm}{dt} = -F \quad (3.1-33)$$

where h is the altitude [ft], r the horizontal distance [NM], m the instantaneous mass of the aircraft [kg] and time t [s] is the independent variable. The thrust T and drag D are in [N], true airspeed V_{TAS} in [kt] and fuel consumption F in [kg/s]. ESF is the energy share factor and $\gamma = dh/dr$ [rad] is the path angle.

In order to perform the integration of the above ODE system, usually, initial values r_0 , h_0 and m_0 for the dependent variables at the initial instant t_0 must be provided. This constitutes the so-called *initial value problem* (IVP).

An alternative way to select one unique solution of the above ODE system is to provide boundary conditions leading to the so-called *boundary value problem* (BVP). As for BEAM scope, only IVP is considered because BVP leads to the much more complex constrained optimization problem whereas the optimization problem associated with IVP remains unconstrained.

3.2. THE COEFFICIENT ESTIMATION PROBLEM

The IVP introduced in the chapter 3.1.4 provides the way to calculate BADA flight profiles once the BADA coefficients are known. Nevertheless, the objective of BEAM is coefficient estimation instead of trajectory generation. In the BEAM problem the unknowns are the BEAM coefficients, which have to be estimated from a set of known *observed* flight profiles.

Since the ODE system is the law that governs trajectory generation according to the BADA model, it can be used for coefficient estimation purposes as well.

In effect, the better the expressions (3.1-31) and (3.1-33) predict the observed derivatives dh/dt and dm/dt , the more accurately the trajectory resulting from the integration of such derivatives will fit the observed one.

Let us assume that the known trajectories for a given aircraft are provided in terms of a set of known altitudes h_i [ft], aircraft mass m_i [Kg], true airspeed TAS_i [kt] and rate of climb ROC_i [fpm] at the time instants t_i , for $i=1\dots n$, n being the number of points representing (sampling) the trajectory. Let also ΔT_{ISA} [K] be the temperature deviation above the standard conditions, for the given trajectory. In order to reproduce the given trajectory by means of BADA trajectory generation model, the right side of the expression:

$$ROC_i = k_4 (T_i - D_i) \frac{TAS_i}{m_i g} ESF_i, \quad (3.2-1)$$

for $i=1\dots n$ must fit as close as possible to the observed ROC_i appearing on the left side, with T_i , D_i and ESF_i calculated as:

$$T_i = t_7 \left(t_0 - t_1 h_i + t_2 \frac{1}{TAS_i} - t_3 \frac{h_i}{TAS_i} + t_4 h_i^2 \right) (t_5 \Delta T_{ISA} + t_6) \quad (3.2-2)$$

$$D_i = \left(d_0 Q_i TAS_i^2 + d_2 \frac{m_i^2}{Q_i TAS_i^2 \cos^2 \gamma_i} \right) (1 + d_{16} M_i^{16}) \quad (3.2-3)$$

$$ESF_i = f(M_i) \quad (3.2-4)$$

$$\gamma_i = \text{asin} \left(\frac{ROC_i}{k_4 TAS_i} \right) \quad (3.2-5)$$

$$M_i = f(TAS_i, h_i, \Delta T_{ISA}) \quad (3.2-6)$$

$$Q_i = f(h_i, \Delta T_{ISA}) \quad (3.2-7)$$

where the functions (3.2-6) and (3.2-7) correspond to the ISA model described in [1], taking into account the ΔT_{ISA} applied to the trajectory in consideration, and (3.2-4) is described in [1] ((3.1-5), (3.1-6), (3.1-7), (3.1-8)).

Expression (3.2-1) represents a measurement process in which left side values are the *observed* values and right side values are the *predicted* values by means of the BADA model.

Mathematically, expression (3.2-1) represents a system of n equations where the unknowns are the BEAM thrust and drag coefficients (at most 13 different ones for thrust and 11 different ones for drag, considering all thrust settings and aerodynamic configurations). Usually, n is greater than the number of thrust and drag coefficients to be estimated, so the system is over determined.

Regarding the expression (3.2-2) and (3.2-3), the character of the expression (3.2-1) is non linear with respect to the BEAM coefficients.

In the same way that expression (3.2-1) governs the evolution of the altitude, expression (3.2-8) predicts the evolution of the aircraft mass:

$$\left(\frac{dm}{dt} \right)_i = -F_i \quad (3.2-8)$$

with:

$$F_i = f_5 [f_0 - f_1 h_i + (f_2 + f_3 TAS_i - f_4 TAS_i^2) T_i] \quad (3.2-9)$$

Since the derivative of m with respect to t at each point i is not expected to be a known value, it has to be estimated by finite differences:

$$\left(\frac{dm}{dt}\right)_i \approx \frac{\Delta m_i}{\Delta t_i} = \frac{m_{i+1} - m_i}{t_{i+1} - t_i} \quad (3.2-10)$$

Therefore, the resulting expression for the mass evolution is:

$$\frac{m_{i+1} - m_i}{t_{i+1} - t_i} = f_5[f_0 - f_1 h_i + (f_2 + f_3 TAS_i - f_4 TAS_i^2)T_i] \text{ for } i=1 \dots n-1 \quad (3.2-11)$$

Again, expression (3.2-11) represents a usually over determined system of $n-1$ equations, non linear with respect to the unknowns, which are the BEAM fuel coefficients (at most 5).

In a strict sense, expressions (3.2-1) and (3.2-11) are coupled. That means that the value of m_i participating in expression (3.2-3) and (3.1-1) should not be the one provided by the observed trajectory, but the predicted by the integration of expression (3.2-11) instead. Also, the value of ROC_i participating in (3.2-5) should be the predicted by (3.2-1) and, therefore, iterations should be needed.

From the previous formulation, the BEAM problem can be characterized as a multivariate non-linear parameter estimation problem, the BEAM coefficients being the parameters to be estimated. Due to the over determined character of the equations, no single set of BEAM parameters will surely verify all the $2n-1$ equations (3.2-1) and (3.2-11). The adopted solution is usually the one that minimizes the sum of square errors (SSE) defined in (3.2-12) for expression (3.2-1):

$$SSE_{ROC} = \sum_{i=1}^n \left[ROC_i - k_4(T_i - D_i) \frac{TAS_i}{m_i g} E_{SF_i} \right]^2 \quad (3.2-12)$$

whereas for expression (3.2-11), SSE is defined as:

$$SSE_{dm/dt} = \sum_{i=1}^{n-1} \left[\frac{m_{i+1} - m_i}{t_{i+1} - t_i} - f_5[f_0 - f_1 h_i + (f_2 + f_3 TAS_i - f_4 TAS_i^2)T_i] \right]^2 \quad (3.2-13)$$

This is the so-called least squares (LS) solution, also known as best solution in the sense of RMS (root-mean square, namely minimum root-mean-square error).

3.2.1. Multiple Trajectory Fitting

The problem can be easily generalized to multiple trajectory fitting. In fact, for a set of m trajectories with respectively $n_1, n_2 \dots n_m$ points each, all the introduced expressions where the index i appears still the same by substituting the index i by i_j ranging as:

$$\begin{aligned} i_1 &= 1 \dots n_1 \\ i_2 &= 1 \dots n_2 \\ &\dots \\ i_m &= 1 \dots n_m \end{aligned}$$

and the number n by the corresponding n_j in each case, with two exceptions:

i) Expressions (3.2-12) and (3.2-13) remain as follows:

$$SSE_{ROC} = \sum_{j=1}^m SSE_{ROC,j} \quad (3.2-14)$$

$$SSE_{dm/dt} = \sum_{j=1}^m SSE_{dm/dt,j} \quad (3.2-15)$$

with $SSE_{ROC,j}$ and $SSE_{dm/dt,j}$ provided respectively for the expressions (3.2-12) and (3.2-13).

ii) ΔT_{ISA} in expressions (3.2-2), (3.2-6), (3.2-7), is $\Delta T_{ISA,j}$ (i.e. may be different for each trajectory).

3.3. MODELLING STRATEGY

As it has been shown, the problem of BEAM coefficient estimation is, in a strict mathematical sense, a coupled multivariate non-linear fitting problem. Nevertheless, the quasi-linear nature of the introduced BEAM formulation of the BADA model, together with the modelling strategy presented here, allow the non-linearity and couple-ness characteristics to be avoided, thus leading to a decomposition of the problem solution as a set of multivariate (decoupled) linear fitting sub-problems.

3.3.1. Available Data

In front of a parameter estimation problem, the right choice of the data used to derive the parameters (reference data) is of the utmost importance. On one hand, the better the reference data represents all the situations in which the model is going to be applied, the better the parameter estimated will allow the model to perform in all such situations. On the other hand, depending on the characteristics of the reference data regarding the characteristics of the model (e.g. allowing decoupling), the complexity of the fitting problem can significantly vary.

Therefore, the following data is supposed to be available for each aircraft for which the BADA model is to be derived:

- 9 climb trajectories $\{CMB_j; j=1\dots9\}$ combining 3 different aircraft masses (maximum, reference and minimum) and 3 different speed profiles (high, reference and low) performed under ISA conditions.
- 3 descent trajectories $\{DES_k; k=1\dots3\}$ for the reference mass and 3 different speed profiles performed under ISA conditions.
- 4 climb trajectories $\{CMB_j; j=10\dots13\}$ combining 3 different aircraft masses and 2 different non-ISA temperature conditions (ISA+10 and ISA+20) performed with the reference speed profile.
- 1 cruise trajectory $\{CRZ\}$ performed under ISA conditions, for the reference mass and speed profile.

For all climb and descent trajectories the position $\{h_i, r_i\}$, speed $\{TAS_i, ROC_i\}$ and mass m_i of the aircraft is assumed to be known at a set of time instants t_i for $i=1\dots n$. For the cruise trajectory only the true airspeed TAS_i and instantaneous fuel consumption F_i at each altitude h_i , are expected to be known.

3.3.2. ISA Climb Trajectory Fitting

ISA climb trajectories are reproduced in BADA with $\Delta T_{ISA}=0$, thrust level set to *max_cmb* and incompressible drag polar (i.e. $C_{M16}=0$) with *clean* aerodynamic configuration. Under these assumptions, expressions (3.2-2) and (3.2-3) are reduced to:

$$T_i = t_0 - t_1 h_i + t_2 \frac{1}{TAS_i} - t_3 \frac{h_i}{TAS_i} + t_4 h_i^2 \quad (3.3-1)$$

$$D_i = d_0 \rho_i TAS_i^2 + d_2 \frac{m_i^2}{\rho_i TAS_i^2 \cos^2 \gamma_i} \quad (3.3-2)$$

In that case, expression (3.2-1) is reduced to an over determined linear system for the estimation of $\{t_0, t_1, t_2, t_3, t_4, d_0, d_2\}$, whose solution that best fits the ROC for all ISA climb trajectories in the sense of RMS is the LS solution obtained with the following elements:

$$\mathbf{H} = \begin{bmatrix} K_1 & K_1 h_1 & \frac{K_1}{TAS_1} & -\frac{K_1 h_1}{TAS_1} & K_1 h_1^2 & -K_1 \rho_1 TAS_1^2 & -\frac{K_1 m_1^2}{\rho_1 TAS_1^2 \cos \gamma_1} \\ K_2 & K_2 h_2 & \frac{K_2}{TAS_2} & -\frac{K_2 h_2}{TAS_2} & K_2 h_2^2 & -K_2 \rho_2 TAS_2^2 & -\frac{K_2 m_2^2}{\rho_2 TAS_2^2 \cos \gamma_2} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ K_n & K_n h_n & \frac{K_n}{TAS_n} & -\frac{K_n h_n}{TAS_n} & K_n h_n^2 & -K_n \rho_n TAS_n^2 & -\frac{K_n m_n^2}{\rho_n TAS_n^2 \cos \gamma_n} \end{bmatrix} \quad (3.3-3)$$

$$K_i = k_4 \frac{TAS_i}{m_i g} E S F_i \quad (3.3-4)$$

$$\mathbf{z} = \begin{bmatrix} ROC_1 \\ ROC_2 \\ \dots \\ ROC_n \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} t_0 \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ d_0 \\ d_2 \end{bmatrix} \quad (3.3-5)$$

Since all \mathbf{c} coefficients are non-negative, the constrained non-negative least squares NNLS can be used instead of the unconstrained LS solution to ensure non-negativity of the coefficients.

3.3.3. ISA Descent Trajectory Fitting

Descent trajectories for modelling purposes are reproduced with $\Delta T_{ISA}=0$, incompressible drag polar (i.e. $C_{M16}=0$) with *clean* aerodynamic configuration and the following thrust level schema:

- *des_hi* for $h \geq h_{des}$
- *des_lo* for $h < h_{des}$

Under these assumptions, expression (3.2-3) remains the same as (3.3-2) and expression (3.2-2) gets reduced to:

$$T_i = t_7 Q_i \tag{3.3-6}$$

with:

$t_7 = C_{Tdes,high}$ for $h \geq h_{des}$ and $t_7 = C_{Tdes,low}$ for $h < h_{des}$

$$Q_i = t_0 - t_1 h_i + t_2 \frac{1}{TAS_i} - t_3 \frac{h_i}{TAS_i} + t_4 h_i^2 \tag{3.3-7}$$

and $\{t_0, t_1, t_2, t_3, t_4\}$ supposed to be known.

In such a case, expression (3.2-1) is reduced to a linear multivariate problem for the estimation of $\{t_7, d_0, d_2\}$ (with two versions of t_7). The solution that overall best fits the ROC for the given trajectories is the LS solution with the following elements:

$$\mathbf{H} = \begin{bmatrix} K_1 Q_1 & -K_1 \rho_1 TAS_1^2 & -\frac{K_1 m_1^2}{\rho_1 TAS_1^2 \cos \gamma_1} \\ K_2 Q_2 & -K_1 \rho_1 TAS_1^2 & -\frac{K_2 m_2^2}{\rho_2 TAS_2^2 \cos \gamma_2} \\ \dots & \dots & \dots \\ K_n Q_n & -K_n \rho_n TAS_n^2 & -\frac{K_n m_n^2}{\rho_n TAS_n^2 \cos \gamma_n} \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} t_7 \\ d_0 \\ d_2 \end{bmatrix} \tag{3.3-8}$$

where K_i are the same as in (3.3-4) and \mathbf{z} is the same as in (3.3-5).

The problem must be split into two similar problems, one for $h \geq h_{des}$ and the other for $h < h_{des}$ so that the two different versions of t_7 can be obtained.

The solution should be constrained to $0 \leq t_7 \leq 1$.

3.3.4. Non-ISA Trajectory Fitting

For $\Delta T_{ISA} \neq 0$, the assumption needed to make the fitting problem linear is that all thrust coefficients are known, with the exception of t_5 and t_6 , which are the ones to be estimated. Incompressible drag polar (i.e. $C_{M16}=0$) and 'clean' aerodynamic configuration are also considered. Under these assumptions, expression (3.1-36) is reduced to:

$$T_i = Q_i(t_6 - t_5 \Delta T_{ISA}) \quad (3.3-9)$$

with $Q_i = t_7 \left(t_0 - t_1 h_i + t_2 \frac{1}{TAS_i} - t_3 \frac{h_i}{TAS_i} + t_4 h_i^2 \right)$ and expression (3.2-3) remains the same as in

(3.3-2). Now, expression (3.2-1) is, again reduced to an over determined linear system, this time for the calculation of $\{t_5, t_6, d_0, d_2\}$, whose solution that best fits the ROC for all provided non-ISA trajectories is the LS solution obtained with the following elements:

$$\mathbf{H} = \begin{bmatrix} K_1 Q_1 \Delta T_{ISA} & K_1 Q_1 & -K_1 Q_1 TAS_1^2 & -\frac{K_1 m_1^2}{Q_1 TAS_1^2 \cos \gamma_1} \\ K_2 Q_2 \Delta T_{ISA} & K_2 Q_2 & -K_2 Q_2 TAS_2^2 & -\frac{K_2 m_2^2}{Q_2 TAS_2^2 \cos \gamma_2} \\ \dots & \dots & \dots & \dots \\ K_n Q_n \Delta T_{ISA} & K_n Q_n & -K_n Q_n TAS_n^2 & -\frac{K_n m_n^2}{Q_n TAS_n^2 \cos \gamma_n} \end{bmatrix} \mathbf{c} = \begin{bmatrix} t_5 \\ t_6 \\ d_0 \\ d_2 \end{bmatrix} \quad (3.3-10)$$

K_i are the same as in (3.3-4) and \mathbf{z} is the same as in (3.3-5).

Temperature constraints from 3.1.1.1 should be applied.

3.3.5. Fuel Consumption Fitting

Provided that the thrust and drag coefficients are known, the thrust T_i can be calculated at each point i of the trajectory, the estimation of the fuel coefficients is always a linear fitting problem for all aircraft types and thrust settings except for *max_crz*, i.e. in cruise, in whose case $f_5=C_{fcr}$.

Climb fuel fitting

For climb (i.e. thrust set to *max_cmb*) $f_0=f_1=0$ and $f_5=1$ for all engine types. Therefore, expression (3.1-44) is reduced to:

$$\frac{m_{i+1} - m_i}{t_{i+1} - t_i} = (f_2 + f_3 TAS_i - f_4 TAS_i^2) T_i \quad (3.3-11)$$

Thus again, leading to an over determined linear system for the estimation of BEAM fuel coefficients $\{f_2, f_3, f_4\}$. This time, the NNLS solution obtained is the one that best fits the mass derivative with respect to the time for the overall climb samples used. That solution is with the following elements:

$$\mathbf{H} = \begin{bmatrix} T_1 & TAS_1 T_1 & -TAS_1^2 T_1 \\ T_2 & TAS_2 T_2 & -TAS_2^2 T_2 \\ \dots & \dots & \dots \\ T_n & TAS_n T_n & -TAS_n^2 T_n \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} f_2 \\ f_3 \\ f_4 \end{bmatrix} \quad \mathbf{z} = \begin{bmatrix} \frac{m_2 - m_1}{t_2 - t_1} \\ \frac{m_3 - m_2}{t_3 - t_2} \\ \dots \\ \frac{m_n - m_{n-1}}{t_n - t_{n-1}} \end{bmatrix} \quad (3.3-12)$$

Cruise fuel fitting

For cruise, $f_0=f_1=0$ for all engine types, f_1, f_2 and f_3 are supposed to be known, so the BEAM fuel coefficient f_5 is the only parameter to be estimated. This time, the problem reduces to a trivial one-dimensional linear fitting problem, being the LS solution that best fits the mass derivative.

$$\mathbf{H} = \begin{bmatrix} (f_2 + f_3 TAS_1 - f_4 TAS_1^2) T_1 \\ (f_2 + f_3 TAS_2 - f_4 TAS_2^2) T_2 \\ \dots \\ (f_2 + f_3 TAS_i - f_4 TAS_i^2) T_i \end{bmatrix} \quad \mathbf{c} = [f_5] \quad \mathbf{z} = \begin{bmatrix} \frac{m_2 - m_1}{t_2 - t_1} \\ \frac{m_3 - m_2}{t_3 - t_2} \\ \dots \\ \frac{m_{i+1} - m_i}{t_{i+1} - t_i} \end{bmatrix} \quad (3.3-13)$$

Descent fuel fitting

For descent $f_2=f_3=f_4=0$ and $f_5=1$ for all engine types. For jet and turboprop, expression (3.2-11) is reduced to:

$$\frac{m_{i+1} - m_i}{t_{i+1} - t_i} = f_0 - f_1 h_i \quad (3.3-14)$$

which is a two-dimensional linear system for the estimation of the BEAM fuel coefficients $\{f_0, f_1\}$. The LS solution, that best fit the derivative of the mass, is obtained with the elements:

$$\mathbf{H} = \begin{bmatrix} 1 & h_1 \\ 1 & h_2 \\ 1 & \dots \\ 1 & h_n \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} f_0 \\ f_1 \end{bmatrix} \quad (3.3-15)$$

and \mathbf{z} the same as in (3.3-12).

3.3.6. Fitting Schema

The proposed overall fitting schema for the estimation of BEAM coefficients takes advantage of the available data for modelling to decouple the problem into a set of linear sub-problems such as the previous described ones. However, it does not limit to find a decoupled sub-optimal solution. Rather, an iterative approach is used, in which as more coefficient estimations become available from the solutions of the decoupled sub-problems, the more new coefficients (as well as the reference data used to estimate them) are taken into account to improve the estimations provided for other different sub-problems. The iterative process continues until no substantial improvements are obtained any more, so the global optimal solution in the sense of RMS is achieved.

This schema can be described in terms of 7 steps as follows:

1. Initialization of the set of trajectories used to estimate $\{t_0, t_1, t_2, t_3, t_4, d_0, d_2, d_{16}\}$ with the ISA climb trajectories $TRJ=\{CMB_j; j=1...9\}$.
2. Estimation of $\{t_0, t_1, t_2, t_3, t_4, d_0, d_2\}$ as described in 3.3.2 from the set TRJ, using as the aircraft mass at each point, m_i , the observed value until estimates of the fuel coefficients $\{f_0, f_1, f_2, f_3, f_4, f_5\}$ become available to allow the predicted value to be used instead.
3. Estimation of $\{t_5, t_6\}$ following 3.3.4 using non ISA climbs. Then addition of the non ISA climb trajectories to the trajectory set so $TRJ=\{CMB_j; j=1...13\}$
4. Estimation of the *hi* and *lo* versions of t_7 as described in 3.3.3 using ISA descent trajectories. Provided that the estimates of $\{t_0, t_1, t_2, t_3, t_4, d_0, d_2\}$ are known from step 1, the *observation matrix* \mathbf{H} in (3.3-8) degenerates to a column vector. On the other hand, since h_{des} is unknown, it has to be determined as the one that makes minimum the corresponding RMS. Then addition of the ISA descent trajectories to the trajectory set so $TRJ=\{CMB_j; j=1...9, DES_k; k=1...3\}$.
5. Now, all the thrust and drag coefficients are estimated. Therefore, the thrust can be calculated for all the points in $\{TRJ, CRZ\}$ and thus, fuel consumption coefficients $\{f_0, f_1, f_2, f_3, f_4, f_5\}$ can be estimated as described in 3.3.5.
6. Repetition from step 1, considering all the trajectories used for modelling with the exception of cruise CRZ (for which, usually only mass evolution is provided), and the BADA predicted values of the aircraft mass instead of the observed ones. Also, step 1 is to solve for the coefficients $\{t_0, t_1, t_2, t_3, t_4, d_0, d_2\}$. Iterations can continue while effectively improving, until the improvements achieved (in terms of RMS_{ROC} and $RMS_{dm/dt}$) falls below a given tolerance.
7. Repetition from step 1 as in step 5 but, this time, for the estimation of $\{t_0, t_1, t_2, t_3, t_4, d_{16}\}$ until the improvements achieved falls below a given tolerance.

The proposed fitting schema is not the unique way to address the optimum estimation of the BEAM coefficients. Other estimation order could be followed as well as other different approaches, like non-linear methods (e.g. *non-linear least squares* method). Nevertheless, from the theoretical point of view, the proposed approach demonstrates that the problem of BADA optimal coefficient estimation can be solved as a set of linear multivariate fitting problems.

3.4. BEAM IDENTIFICATION TOOLBOX

The BEAM identification toolbox is a Matlab toolbox developed for the identification purposes. The BEAM implementation is a part of the BEAM identification toolbox. It consists of several toolboxes:

- general computing - provides mathematical and physical functions required by all other toolboxes;
- earth model - models the environment in which the aircraft movement takes place;
- BEAM Identification - identifies the BADA 3.7 Aircraft Performance Model coefficients;
- BADA 3.7 Aircraft Performance Model (APM) - models the aircraft performances based on the BADA 3.7 Aircraft Performance Model;
- two dimensional trajectory computation infrastructure - integrates trajectories in two dimensions based on the Earth model, the APM, and certain intent instructions.

3.5. BEAM IDENTIFICATION PROCESS

The Beam identification process is an iterative process based on the BEAM identification toolbox. It should be performed for each aircraft separately and may be divided in several parts:

- input data set (selection of the input data set, trajectories);
- identification process (adjustment of the identification parameters);
- results analysis.

The identification is executed on the selected input set of data and results are checked. According to the results the identification parameters and input data set are adjusted in order to get the best possible results. This procedure is repeated until the optimal (at least acceptable) results are obtained. Simplified BEAM identification process is shown in Figure 3-2. The BEAM identification process for an aircraft results in the BADA model for that aircraft. As mentioned above, the BADA model consists of OPF file and APF file.

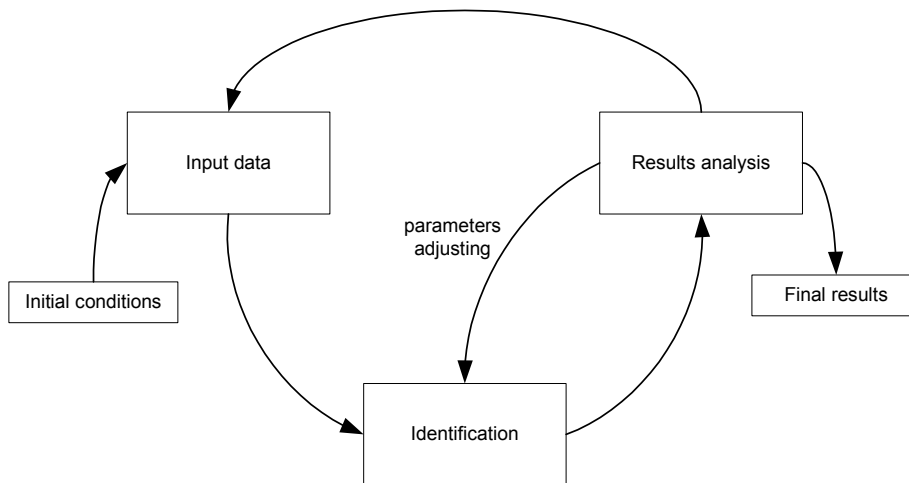


Figure 3-2: BEAM identification process

3.5.1. Identification Input Data Set

The identification input data set is used for the identification of the BADA model.

It consists of:

- technical specification data;
- default airline procedure speed schedule data;
- buffet capability data (optional);
- altitude capability data (optional);
- performance data (trajectories).

By selection of an input data set, the selection of performance data (trajectories) is considered. All other parameters during the identification process should be left unchanged. The performance data used in the identification process are based on the optimal recommended set of performance data, provided in chapter 3.3.1, and available data (provided by the manufacturers).

3.5.2. Identification Process

The identification process itself will be explained here. The output of the identification process is a BADA model. It may be divided in several steps:

- data smoothing – the identification process provides the user with the possibility of smoothing the input data to obtain ROC values that better resemble those experienced by the aircraft.
- importing data – the identification process imports aircraft characteristics from the identification input data set.

- main identification – a user is able to choose among four options: normal identification, normal identification positive coefficients, define C_{d0} range and define C_{d0} range positive coefficients. For pistons just two options are available. This helps a user to impose particular constraints to all or some of the models' coefficients, assisting in the decoupling of the effects of thrust and drag.
- comparison with BADA atmosphere - the identification process employs the new atmospheric model, together with the new Energy Share Factor definitions, which are different from those described in [1]. In particular, the atmospheric model employed in this toolset contains new physical dependencies that result in different temperature and pressure profiles with altitude for non standard atmospheres, while the Energy Share Factors definitions add corrections for non-standard conditions that were not present in the [1] definitions. Note that just the identification is done with the new model while the Matlab implementation is done with the old model.
- results obtained and presented during the identification process:
 - iterations made in h_{des} together with the error metric obtained;
 - global rate of climb and fuel consumption errors (RMS) achieved with the identification process;
 - description of all the trajectories employed in the identification process;
 - rate of climb identification results for each trajectory;
 - fuel consumption identification results for each trajectory;
 - identified drag, thrust, and fuel consumption BEAM coefficients;
 - identified drag, thrust, and fuel consumption BADA coefficients.
- text file generation – at some point AC___.opf, AC___.apf files are created (if requested);
- trajectory comparison – at the end of the identification process a user may request trajectory comparison which compares the original trajectories with replicas computed with the identified models.

A user may choose among 5 different smooth levels and four different identification types as mention above. During the identification all combinations of identification parameters should be tested.

3.5.3. Results Analysis

The identification results are structured in files that contain all needed data. They consist of parameters identified in the identification process or parameters directly taken from manufacturers' manuals for each aircraft. The result files for each aircraft are:

- AC.xml – aircraft parameters;
- AC___.apf – airline performance data contains speed schedule for climb, cruise and descent phases;
- AC___.opf – operations performance file that contains performance parameters.

Obtained results are analyzed in terms of the accuracy of the identified model regarding to the reference data. The trajectories, computed using the identified model, (computed trajectories) were compared with the trajectories used in the identification process (referent trajectories) for an aircraft model. Again, the identification is done using new atmosphere model, while the result

analysis is done for both atmosphere models.

The accuracy for the new ISA model is presented in terms of RMS, MEAN, STD, MAX values for both, vertical speed and fuel flow. RMS, MEAN, STD and MAX are defined for errors (differences) between computed trajectories and referent trajectories. Similarly, the accuracy for the old ISA model is presented in terms of RMS, MEAN, STD, MAX values for both, vertical speed and fuel flow. Table 3-11 shows an example of errors vertical speed for an aircraft. The detailed accuracy report for all modelled aircrafts is provided in [3].

Table 3-11: Example of vertical speed errors

| TRJ ID | TRJ Type | CAS <FL100 | CAS >FL100 | Ma | Aircraft mass | Delta ISA | RMS [ft/min] | MEAN [ft/min] | STD [ft/min] | MAX [ft/min] |
|--------|----------|------------|------------|------|---------------|-----------|--------------|---------------|--------------|--------------|
| 1 | MCMB | 310 | 310 | 0.78 | 45000 | 0 | 96.07742 | -29.59718 | 91.40503 | 180.24462 |
| 2 | MCMB | 310 | 310 | 0.78 | 60000 | 0 | 66.27394 | -4.73766 | 66.10438 | -249.27347 |
| 3 | MCMB | 310 | 310 | 0.78 | 70000 | 0 | 58.2253 | -10.64105 | 57.24469 | -266.18087 |
| 4 | MCMB | 250 | 250 | 0.6 | 45000 | 0 | 228.3291 | 75.06287 | 215.63799 | -458.15377 |
| 5 | MCMB | 250 | 250 | 0.6 | 60000 | 0 | 163.05745 | 50.57795 | 155.01485 | -325.37007 |
| 6 | MCMB | 250 | 250 | 0.6 | 70000 | 0 | 126.70266 | 16.12381 | 125.67254 | -251.10932 |
| 7 | MCMB | 340 | 340 | 0.78 | 45000 | 0 | 173.66219 | -96.90881 | 144.10843 | -338.23075 |
| 8 | MCMB | 340 | 340 | 0.78 | 60000 | 0 | 105.70821 | -51.66308 | 92.22338 | -246.76213 |
| 9 | MCMB | 340 | 340 | 0.78 | 70000 | 0 | 80.1794 | -42.52073 | 67.9759 | -263.16907 |
| 10 | MCMB | 280 | 280 | 0.76 | 45000 | 0 | 108.92569 | 62.65697 | 89.10056 | 235.20504 |
| 11 | MCMB | 280 | 280 | 0.76 | 60000 | 0 | 96.42343 | 40.66345 | 87.42975 | -265.29215 |
| 12 | MCMB | 280 | 280 | 0.76 | 70000 | 0 | 84.56154 | 8.44674 | 84.13862 | -274.48794 |
| 13 | MCMB | 310 | 310 | 0.78 | 45000 | 10 | 88.5208 | -8.35072 | 88.12603 | 196.05099 |
| 14 | MCMB | 310 | 310 | 0.78 | 60000 | 10 | 67.38551 | 11.63869 | 66.37279 | -245.19984 |
| 15 | MCMB | 310 | 310 | 0.78 | 70000 | 10 | 59.10487 | 3.7925 | 58.98307 | -262.44868 |
| 16 | MCMB | 310 | 310 | 0.78 | 60000 | 20 | 48.22811 | 2.29922 | 48.17327 | -112.58197 |
| 17 | LIDL | 300 | 300 | 0.78 | 60000 | 0 | 50.22349 | 38.36642 | 32.41014 | 88.9561 |
| 18 | LIDL | 340 | 340 | 0.8 | 60000 | 0 | 88.90037 | -71.81957 | 52.39489 | -144.92407 |
| 19 | LIDL | 250 | 250 | 0.76 | 60000 | 0 | 76.77015 | 54.0622 | 54.50628 | 132.25602 |

In order to get better overview graphs with reference and computed trajectories are used in analysis for all input trajectories for each aircraft. In general, four different graph types are used: geopotential pressure altitude as function of time, vertical speed as function of geopotential altitude, aircraft mass as function of time and geopotential pressure altitude as function of horizontal distance. An example of the graph containing just one trajectory is shown on the Figure 3-3.

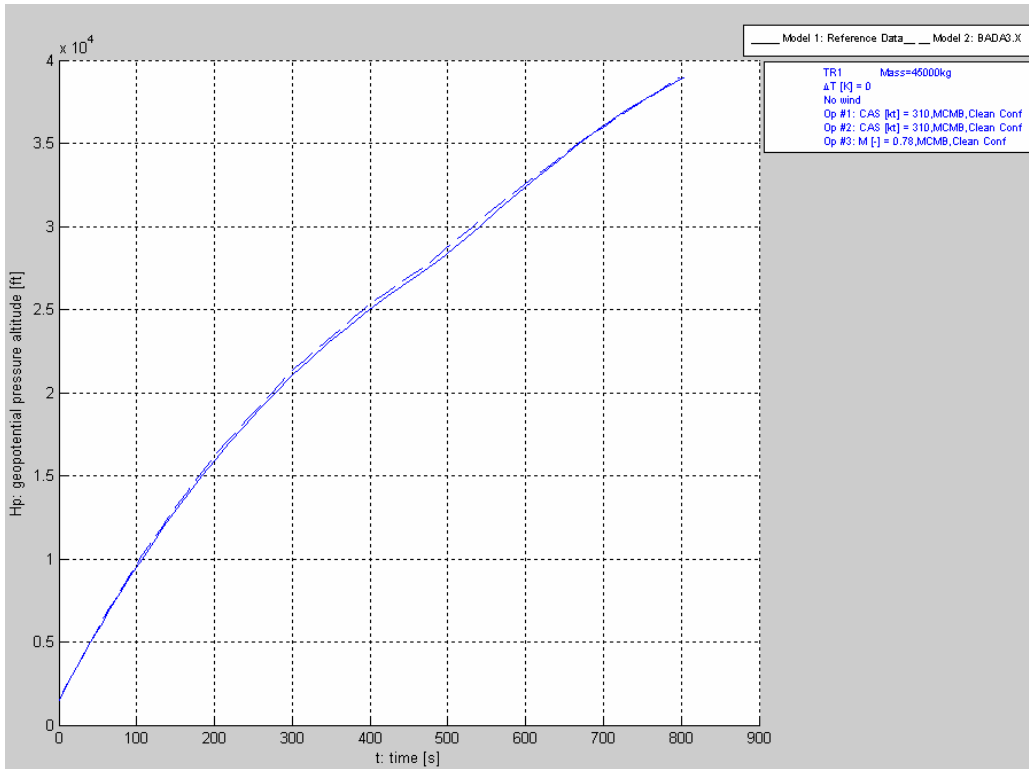


Figure 3-3: Geopotential pressure altitude as function of time

3.6. BEAM IMPLEMENTATION

At the end of the BEAM identification process the BEAM implementation must be executed. The BEAM implementation is the baseline BADA implementation. It follows the guidelines provided in the BADA user manual [1]. It uses the results from the identification process (AC__.opf and AC__.apf) and generates:

- performance table file, AC__.ptf;
- performance table data file, AC__.ptd (performance table data with more details, for more details see [1]);
- performance table extended file, AC__.xtf – for internal use only;
- performance table data extended file with more details, AC__.xdf – for internal use only.

The BEAM implementation diagram is depicted in Figure 3-4. Performance table files (PTF files) and performance table data files (PTD) are provided as part of the release files.

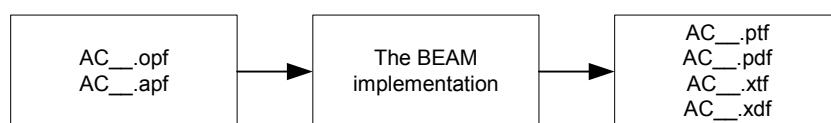


Figure 3-4: The BEAM implementation diagram

4. VALIDATION PROCESS

The validation process includes checking the behaviour of developed models in respect to input performance data, checking the syntax of release files, cross validation of the BEAM implementation and validation of developed models in respect to real data. The first three parts are performed for all developed models while the fourth part depends of available real data.

A part of the validation process comparing the behaviour of the identified models with input reference data is already included in the identification process described above. The validation is done in terms of errors and graphically as described in chapter 3.5.3. It is assumed that good matching with the input performance data is obtained through the identification iterative process.

4.1. SYNTAX VALIDATION

As part of validation process the syntaxes of release files are checked. This should ensure that all generated files follow definitions provided in [1]. When possible, parameter ranges are checked too.

4.2. CROSS VALIDATION

The cross validation is used for validation of different BADA implementations using identified aircraft models. The BADA web calculation tool is a BADA implementation that is available to various users and the behaviour of new models in this tool should be tested. The BEAM implementation and web calculation tool implementation are compared for climb and descent phases using the developed comparison tool. It uses AC__.ptf, AC__.opf files for the BEAM implementation and AC__CL.txt, AC_DE.txt (output files for web calculation tool for climb and descent phases) files for the web calculation tool. The TAS (true airspeed), ROCD (rate of climb or descent) and FF (fuel flow) values are validated for climb and descent phases for all FL (flight levels) for all aircrafts. For each aircraft for each FL in both phases the TAS error is defined as:

$$TAS_{Error} = \frac{TAS_{Matlab} - TAS_{Web}}{TAS_{Matlab}} \times 100 (\%) \quad (4.2-1)$$

If absolute value of the TAS_{Error} is greater or equal to 1%, this is reported as an error in TAS for a particular aircraft on a particular FL in one of phases, i.e. alert criteria is 1 %.

For each aircraft for each FL in both phases the ROCD error is defined as:

$$ROCD_{Error} = \frac{ROCD_{Matlab} - ROCD_{Web}}{ROCD_{Matlab}} \times 100 (\%) \quad (4.2-2)$$

If absolute value of the ROCD_{Error} is greater or equal to 10%, this is reported as an error in ROCD. If the absolute value of the ROCD_{Error} is greater or equal to 5 and less than 10, this is reported as a ROCD warning.

For each aircraft for each FL in both phases the FF error is defined as:

$$FFError = \frac{FF_{Matlab} - FF_{Web}}{FF_{Matlab}} \times 100 (\%) \tag{4.2-3}$$

If the absolute value of the FFError is greater or equal to 10%, this is reported as an error in FF. If the absolute value of the FFError is greater or equal to 5 and less than 10 that is considered and reported as a FF warning.

All implementation conflicts, errors and warnings must be resolved.

4.3. RDAP VALIDATION

The RDAP validation is used in order to validate the behaviour of identified models in respect to real data. Moreover, it should help in testing if allowed parameters' changes may satisfy specific local needs, e.g. if proposed climb speed range may improve local performance. The RDAP validation is marked yellow on Figure 1-1 because it is not mandatory and it is highly dependent on operational data availability. It is expected that in the future this validation part will be included as part of the BADA APM release process. It will be based on the RDAP tool and one of the EUROCONTROL implementations. The overall RDAP architecture is shown in Figure 4-1.

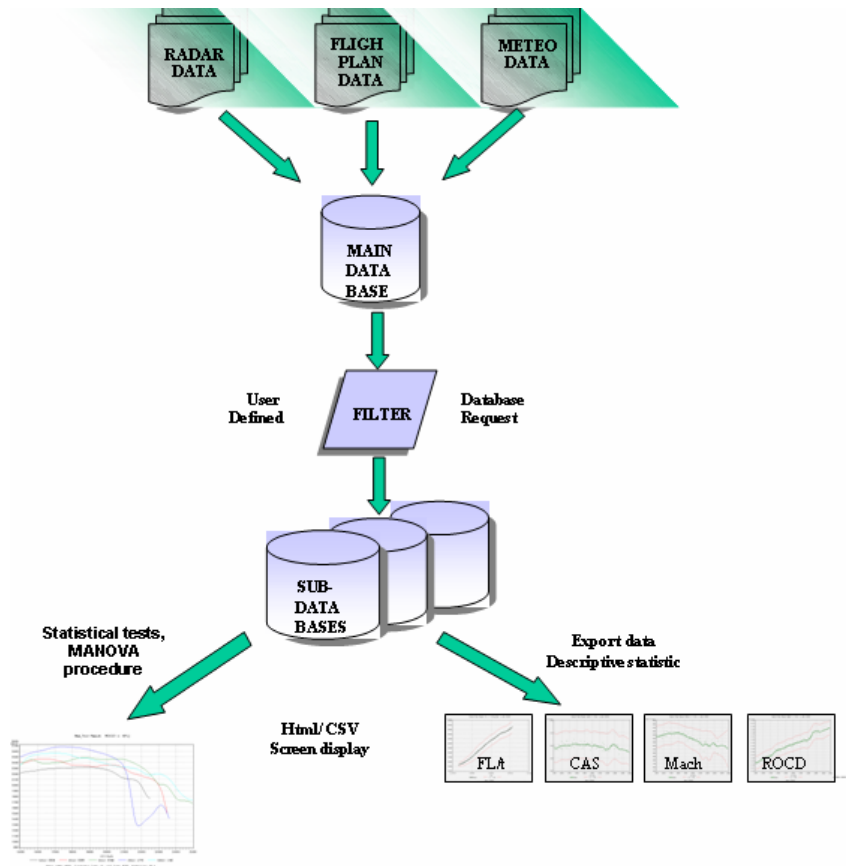


Figure 4-1: RDAP overall architecture

In general, appropriate real trajectories from RDAP tool should be identified, extracted and compared with calculated trajectories generated using new models.

5. RESULTS

At the end the results from the BADA aircraft performance modelling process will be described. The results are based on the identification and validation process. End results consist of documentation and release files that contain all needed parameters. The documentation consists of:

- modelling and accuracy reports – for each aircraft modelling the accuracy report must be generated. It contains all aircraft and identification details as well as the identification results. This document is intended for internal use only.
- modelling report summary report – basically, this document contains two accuracy table for each aircraft. Both tables contain RMS (root mean square) error values, MEAN error values, STD (standard deviation) values and MAX error values for the ROCD for all climb and descent trajectories used in the identification process. They contain the same values for fuel flow for climb, descent and cruise trajectories used in the identification process. The first table was obtained using the new atmosphere model while the second one was obtained using the old atmosphere model. This document will be distributed to the users.

For each modelled aircraft the release files, which will be distributed to the users, are:

- AC__.apf – airline performance data contains speed schedule for climb, cruise and descent phases.
- AC__.opf – operations performance file that contains performance parameters;
- AC__.ptf – an implementation file (example), contains summary performance tables for climb, cruise and descent phases.
- AC__.ptd – extended implementation files, contains more implementation details (different masses in climb, intermediate parameters).

Operation performance parameters summary and airlines performance speed schedules summary, with sources, are given in the Appendix B.

Details about files' contents and formats may be found in [1].

Page intentionally left blank

6. APPENDIX A

This appendix contains examples of row input data.

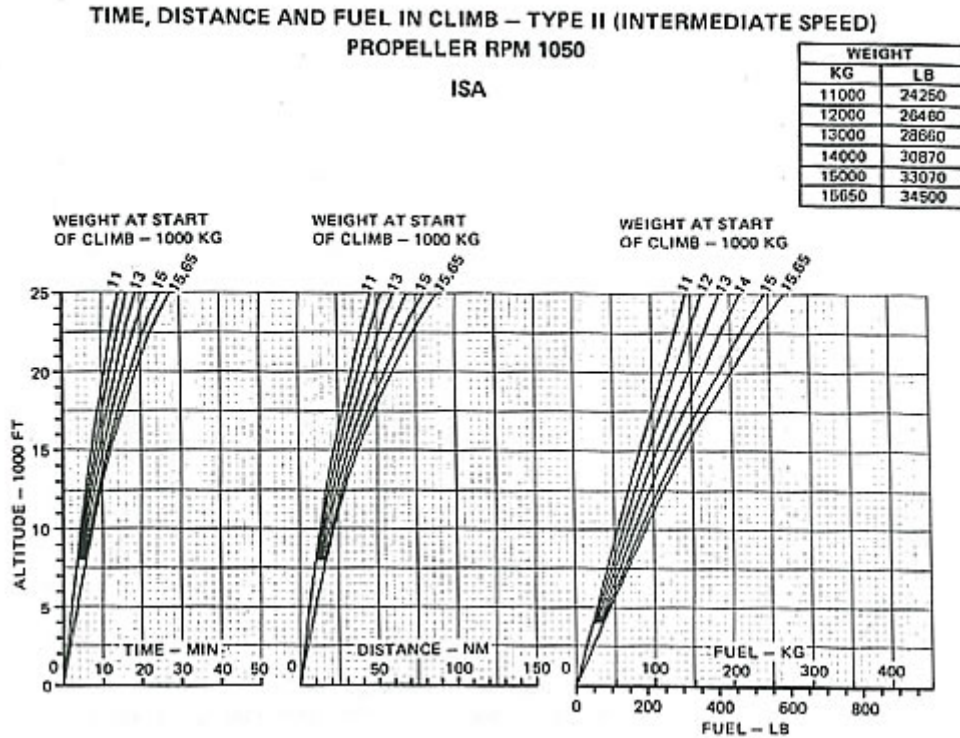


Figure 6-1: Time, distance and fuel in a climb phase

CLIMB 280/0.78

| ALT. | ALTG | WGHT | MACH | CAS | TAS | WIND | DIST | TIME | FUEL | RATE | GRDT | ALPH | CL | CD | WFE | FN | NI |
|-------|-------|--------|-------|-------|-------|------|-------|-------|------|--------|--------|--------|---------|---------|--------|-------|--------|
| (FT) | (FT) | (KG) | () | (KT) | (KT) | (KT) | (NM) | (MN) | (KG) | (FTMN) | (DEG.) | (DEG.) | () | () | (KG/H) | (DAN) | (%) |
| 41000 | 41000 | 148983 | 0.78 | 230.1 | 447.4 | 0 | 111.5 | 16.63 | 3017 | 861.7 | 1.09 | 2.96 | 0.51901 | 0.02646 | 6252 | 10060 | 98.604 |
| 40000 | 40000 | 147097 | 0.78 | 235.5 | 447.4 | 0 | 103.5 | 16.56 | 2903 | 1005.6 | 1.27 | 2.78 | 0.49506 | 0.02539 | 6556 | 10565 | 98.673 |
| 39000 | 39000 | 147201 | 0.78 | 241 | 447.4 | 0 | 96.6 | 14.64 | 2799 | 1149.7 | 1.45 | 2.61 | 0.47218 | 0.02442 | 6870 | 11091 | 98.715 |
| 38000 | 38000 | 147297 | 0.78 | 246.7 | 447.4 | 0 | 90.5 | 13.82 | 2703 | 1289.2 | 1.63 | 2.45 | 0.45033 | 0.02361 | 7199 | 11643 | 98.753 |
| 37000 | 37000 | 147388 | 0.78 | 252.5 | 447.4 | 0 | 85.1 | 13.08 | 2612 | 1431 | 1.81 | 2.29 | 0.42947 | 0.02288 | 7544 | 12222 | 98.786 |
| 36000 | 36000 | 147473 | 0.78 | 258.4 | 447.6 | 0 | 80.1 | 12.42 | 2527 | 1712 | 2.16 | 2.14 | 0.40952 | 0.02224 | 7908 | 12829 | 98.847 |
| 35000 | 35000 | 147549 | 0.78 | 264.4 | 449.6 | 0 | 75.9 | 11.85 | 2451 | 1846.5 | 2.32 | 2 | 0.39063 | 0.02172 | 8271 | 13388 | 98.795 |
| 34000 | 34000 | 147623 | 0.78 | 270.5 | 451.6 | 0 | 71.9 | 11.33 | 2377 | 1978.3 | 2.48 | 1.87 | 0.37276 | 0.02126 | 8643 | 13959 | 98.732 |
| 33000 | 33000 | 147695 | 0.78 | 276.7 | 453.7 | 0 | 68.3 | 10.84 | 2305 | 2108 | 2.63 | 1.74 | 0.35586 | 0.02086 | 9023 | 14544 | 98.652 |
| 32000 | 32000 | 147778 | 0.773 | 280 | 451.4 | 0 | 64.1 | 10.3 | 2222 | 2610.4 | 2.02 | 1.7 | 0.34645 | 0.0206 | 9370 | 15127 | 98.582 |
| 31000 | 31000 | 147873 | 0.757 | 280 | 444.3 | 0 | 59.7 | 9.7 | 2127 | 1741.3 | 2.22 | 1.74 | 0.34452 | 0.02046 | 9685 | 15715 | 98.613 |
| 30000 | 30000 | 147964 | 0.742 | 280 | 437.4 | 0 | 55.6 | 9.15 | 2036 | 1861.7 | 2.41 | 1.76 | 0.34272 | 0.02035 | 9989 | 16283 | 98.761 |
| 29000 | 29000 | 148052 | 0.727 | 280 | 430.5 | 0 | 51.8 | 8.62 | 1948 | 1961.5 | 2.58 | 1.77 | 0.341 | 0.02025 | 10222 | 16775 | 98.513 |
| 28000 | 28000 | 148138 | 0.713 | 280 | 423.9 | 0 | 48.3 | 8.13 | 1862 | 2062.9 | 2.75 | 1.79 | 0.33936 | 0.02015 | 10462 | 17281 | 98.279 |
| 27000 | 27000 | 148221 | 0.699 | 280 | 417.3 | 0 | 45 | 7.65 | 1779 | 2165.9 | 2.94 | 1.8 | 0.33778 | 0.02005 | 10711 | 17805 | 98.052 |
| 26000 | 26000 | 148303 | 0.685 | 280 | 410.8 | 0 | 41.9 | 7.2 | 1697 | 2270.6 | 3.13 | 1.82 | 0.33626 | 0.01997 | 10968 | 18352 | 97.779 |
| 25000 | 25000 | 148382 | 0.672 | 280 | 404.5 | 0 | 39 | 6.77 | 1618 | 2381.5 | 3.33 | 1.83 | 0.3348 | 0.0199 | 11249 | 18935 | 97.582 |
| 24000 | 24000 | 148460 | 0.659 | 280 | 398.3 | 0 | 36.2 | 6.36 | 1540 | 2507.8 | 3.56 | 1.85 | 0.33337 | 0.01983 | 11588 | 19598 | 97.582 |
| 23000 | 23000 | 148536 | 0.646 | 280 | 392.2 | 0 | 33.7 | 5.98 | 1464 | 2635 | 3.8 | 1.87 | 0.332 | 0.01975 | 11931 | 20276 | 97.555 |
| 22000 | 22000 | 148611 | 0.634 | 280 | 386.2 | 0 | 31.3 | 5.6 | 1389 | 2763.5 | 4.05 | 1.88 | 0.33067 | 0.01969 | 12275 | 20974 | 97.501 |
| 21000 | 21000 | 148684 | 0.622 | 280 | 380.3 | 0 | 29 | 5.25 | 1316 | 2891 | 4.3 | 1.9 | 0.32938 | 0.01962 | 12620 | 21681 | 97.42 |
| 20000 | 20000 | 148757 | 0.61 | 280 | 374.6 | 0 | 26.9 | 4.91 | 1243 | 3013.6 | 4.56 | 1.92 | 0.32814 | 0.01956 | 12956 | 22379 | 97.31 |
| 19000 | 19000 | 148828 | 0.598 | 280 | 368.9 | 0 | 24.9 | 4.59 | 1172 | 3119.1 | 4.79 | 1.93 | 0.32697 | 0.01951 | 13244 | 23017 | 97.008 |
| 18000 | 18000 | 148898 | 0.587 | 280 | 363.4 | 0 | 23 | 4.27 | 1102 | 3225 | 5.03 | 1.92 | 0.32587 | 0.01952 | 13530 | 23696 | 96.62 |
| 17000 | 17000 | 148968 | 0.576 | 280 | 358 | 0 | 21.1 | 3.96 | 1032 | 3298.8 | 5.22 | 1.92 | 0.32485 | 0.01954 | 13713 | 24240 | 95.902 |
| 16000 | 16000 | 149037 | 0.565 | 280 | 352.7 | 0 | 19.4 | 3.66 | 963 | 3357.3 | 5.39 | 1.91 | 0.32389 | 0.01957 | 13864 | 24722 | 95.105 |
| 15000 | 15000 | 149106 | 0.555 | 280 | 347.4 | 0 | 17.6 | 3.37 | 894 | 3395.9 | 5.54 | 1.9 | 0.32299 | 0.0196 | 13972 | 25116 | 94.225 |
| 14000 | 14000 | 149174 | 0.544 | 280 | 342.3 | 0 | 16 | 3.08 | 826 | 3500 | 5.79 | 1.89 | 0.32203 | 0.0196 | 14279 | 25823 | 93.922 |
| 13000 | 13000 | 149242 | 0.534 | 280 | 337.3 | 0 | 14.4 | 2.8 | 758 | 3599.1 | 6.05 | 1.89 | 0.32112 | 0.01957 | 14578 | 26508 | 93.591 |
| 12000 | 12000 | 149309 | 0.525 | 280 | 332.3 | 0 | 12.9 | 2.52 | 691 | 3693.7 | 6.3 | 1.88 | 0.32023 | 0.01955 | 14868 | 27184 | 93.229 |
| 11000 | 11000 | 149376 | 0.515 | 280 | 327.5 | 0 | 11.4 | 2.25 | 624 | 3783.3 | 6.55 | 1.87 | 0.31937 | 0.01952 | 15148 | 27848 | 92.833 |
| 10000 | 10000 | 149442 | 0.506 | 280 | 322.8 | 0 | 10 | 1.99 | 558 | 3841.5 | 6.75 | 1.86 | 0.31859 | 0.01949 | 15351 | 28361 | 92.404 |
| 9000 | 9000 | 149509 | 0.497 | 280 | 318.1 | 0 | 8.6 | 1.74 | 491 | 3954 | 7.05 | 1.86 | 0.31774 | 0.01946 | 15709 | 29159 | 92.282 |
| 8000 | 8000 | 149575 | 0.488 | 280 | 313.5 | 0 | 7.3 | 1.49 | 425 | 4056.6 | 7.35 | 1.85 | 0.31692 | 0.01942 | 16046 | 29938 | 92.117 |
| 7000 | 7000 | 149641 | 0.479 | 280 | 309.1 | 0 | 6.1 | 1.24 | 359 | 4160.2 | 7.64 | 1.84 | 0.31612 | 0.01938 | 16375 | 30705 | 91.931 |
| 6000 | 6000 | 149706 | 0.47 | 280 | 304.7 | 0 | 4.9 | 1 | 294 | 4253.7 | 7.92 | 1.84 | 0.31535 | 0.01935 | 16710 | 31449 | 91.722 |
| 5000 | 5000 | 149772 | 0.462 | 280 | 300.4 | 0 | 3.7 | 0.77 | 228 | 4338.9 | 8.2 | 1.83 | 0.31461 | 0.01931 | 17037 | 32162 | 91.482 |
| 4000 | 4000 | 149837 | 0.454 | 280 | 296.1 | 0 | 2.6 | 0.54 | 163 | 4453.1 | 8.54 | 1.82 | 0.31382 | 0.01927 | 17451 | 33046 | 91.389 |
| 3000 | 3000 | 149902 | 0.446 | 280 | 292 | 0 | 1.5 | 0.32 | 98 | 4565.9 | 8.88 | 1.82 | 0.31304 | 0.01923 | 17863 | 33941 | 91.269 |
| 2000 | 2000 | 149967 | 0.438 | 280 | 287.9 | 0 | 0.5 | 0.11 | 33 | 4676.7 | 9.23 | 1.81 | 0.31227 | 0.0192 | 18272 | 34844 | 91.123 |
| 1500 | 1500 | 150000 | 0.434 | 280 | 285.9 | 0 | 0 | 0 | 0 | 4728.3 | 9.4 | 1.8 | 0.31189 | 0.01918 | 18467 | 35281 | 91.04 |

Figure 6-2: Parameters in a climb phase

| | | FLIGHT PLANNING Climb Planning | | | | 03-06-16 Jun 21/92 | | | | | | |
|--|-------|-----------------------------------|-------|------|-------|-----------------------|-------|----------|-------|------|-------|------|
| CLIMB 250 / 290 KIAS / 0.74 M | | | | | | | | | | | | |
| MAX. CLIMB THRUST NORMAL ACU's A/I OFF | | ISA 25% C.G. | | | | FROM 1500 FT | | | | | | |
| | | | | | | TIME(MIN) | | FUEL(KG) | | | | |
| FLIGHT LEVEL | | INITIAL CLIMB WEIGHT - 1000 KG | | | | | | | | | | |
| | | 13 | | 14 | | 15 | | 16 | | 17 | | |
| 410 | 15.7 | 418 | 17.4 | 460 | 19.3 | 505 | 21.5 | 556 | 24.2 | 616 | 27.7 | 691 |
| | 103.9 | 397 | 115.3 | 398 | 128.2 | 399 | 143.3 | 400 | 161.8 | 401 | 186.4 | 403 |
| | 0.74 | 907 | 0.74 | 754 | 0.74 | 614 | 0.74 | 485 | 0.74 | 360 | 0.74 | 238 |
| 390 | 13.8 | 388 | 15.1 | 424 | 16.6 | 462 | 18.2 | 504 | 20.1 | 550 | 22.2 | 602 |
| | 90.4 | 393 | 99.4 | 394 | 109.3 | 395 | 120.3 | 396 | 132.7 | 397 | 147.1 | 398 |
| | 0.74 | 1209 | 0.74 | 1044 | 0.74 | 894 | 0.74 | 757 | 0.74 | 631 | 0.74 | 510 |
| 370 | 12.3 | 361 | 13.4 | 393 | 14.6 | 427 | 15.9 | 463 | 17.4 | 502 | 18.9 | 545 |
| | 79.8 | 389 | 87.3 | 390 | 95.3 | 391 | 104 | 392 | 113.5 | 392 | 124.2 | 393 |
| | 0.74 | 1459 | 0.74 | 1287 | 0.74 | 1131 | 0.74 | 987 | 0.74 | 855 | 0.74 | 734 |
| 350 | 11.1 | 338 | 12.1 | 367 | 13.1 | 397 | 14.2 | 429 | 15.4 | 464 | 16.6 | 501 |
| | 71.2 | 386 | 77.6 | 386 | 84.3 | 387 | 91.5 | 387 | 99.3 | 388 | 107.8 | 389 |
| | 0.74 | 1814 | 0.74 | 1624 | 0.74 | 1451 | 0.74 | 1291 | 0.74 | 1144 | 0.74 | 1008 |
| 330 | 10 | 316 | 10.9 | 343 | 11.8 | 370 | 12.7 | 399 | 13.8 | 430 | 14.8 | 463 |
| | 63.8 | 381 | 69.3 | 382 | 75.1 | 382 | 81.3 | 383 | 87.9 | 383 | 94.9 | 384 |
| | 0.74 | 2038 | 0.74 | 1844 | 0.74 | 1667 | 0.74 | 1505 | 0.74 | 1354 | 0.74 | 1215 |
| 310 | 9.1 | 294 | 9.8 | 319 | 10.6 | 344 | 11.5 | 370 | 12.3 | 398 | 13.3 | 427 |
| | 56.9 | 376 | 61.8 | 376 | 66.8 | 377 | 72.1 | 377 | 77.7 | 378 | 83.7 | 378 |
| | 0.74 | 2158 | 0.74 | 1966 | 0.74 | 1791 | 0.74 | 1633 | 0.74 | 1484 | 0.74 | 1347 |
| 290 | 8.2 | 272 | 8.9 | 294 | 9.6 | 317 | 10.3 | 341 | 11 | 366 | 11.8 | 392 |
| | 50.3 | 369 | 54.6 | 370 | 58.9 | 370 | 63.5 | 371 | 68.3 | 371 | 73.3 | 372 |
| | 0.74 | 2239 | 0.74 | 2050 | 0.74 | 1879 | 0.74 | 1723 | 0.74 | 1580 | 0.74 | 1445 |
| 270 | 7.1 | 245 | 7.7 | 265 | 8.3 | 285 | 8.9 | 306 | 9.6 | 328 | 10.3 | 351 |
| | 42.8 | 360 | 46.3 | 360 | 50 | 360 | 53.8 | 361 | 57.7 | 361 | 61.8 | 362 |
| | 0.722 | 1883 | 0.722 | 1729 | 0.722 | 1592 | 0.722 | 1467 | 0.722 | 1353 | 0.722 | 1248 |
| 250 | 6.2 | 218 | 6.7 | 236 | 7.2 | 254 | 7.7 | 272 | 8.2 | 291 | 8.8 | 311 |
| | 35.9 | 349 | 38.9 | 350 | 41.9 | 350 | 45 | 350 | 48.2 | 351 | 51.5 | 351 |
| | 0.694 | 2247 | 0.694 | 2070 | 0.694 | 1911 | 0.694 | 1769 | 0.694 | 1638 | 0.694 | 1519 |
| 200 | 4.3 | 161 | 4.6 | 174 | 4.9 | 187 | 5.3 | 201 | 5.7 | 214 | 6 | 228 |
| | 23.1 | 326 | 25 | 326 | 26.9 | 326 | 28.8 | 327 | 30.8 | 327 | 32.9 | 327 |
| | 0.631 | 3017 | 0.631 | 2790 | 0.631 | 2591 | 0.631 | 2413 | 0.631 | 2252 | 0.631 | 2105 |
| 150 | 2.8 | 113 | 3 | 122 | 3.3 | 131 | 3.5 | 141 | 3.7 | 150 | 4 | 160 |
| | 14.1 | 302 | 15.3 | 302 | 16.5 | 303 | 17.6 | 303 | 18.9 | 303 | 20.1 | 304 |
| | 0.574 | 3937 | 0.574 | 3648 | 0.574 | 3395 | 0.574 | 3170 | 0.574 | 2970 | 0.574 | 2787 |
| 100 | 1.4 | 61 | 1.6 | 66 | 1.7 | 71 | 1.8 | 76 | 1.9 | 82 | 2 | 87 |
| | 6.4 | 266 | 6.9 | 267 | 7.5 | 267 | 8 | 268 | 8.6 | 269 | 9.1 | 269 |
| | 0.452 | 5328 | 0.452 | 4932 | 0.452 | 4583 | 0.452 | 4274 | 0.452 | 3996 | 0.452 | 3747 |
| 50 | 0.6 | 25 | 0.6 | 27 | 0.7 | 29 | 0.7 | 31 | 0.7 | 33 | 0.8 | 36 |
| | 2.4 | 255 | 2.6 | 256 | 2.8 | 256 | 3 | 257 | 3.2 | 258 | 3.4 | 258 |
| | 0.413 | 6038 | 0.413 | 5593 | 0.413 | 5203 | 0.413 | 4857 | 0.413 | 4548 | 0.413 | 4270 |

CLB29001 - 17/04/92
Climb Performance (Climb Speed Schedule 250/290 KIAS/0.74 M) - ISA (page 1 of 2)
Figure 03-06-8

| | |
|--|--|
| Flight Planning and Cruise Control Manual CSP A-015 | |
|--|--|

Figure 6-3: Time, distance, fuel in a climb phase

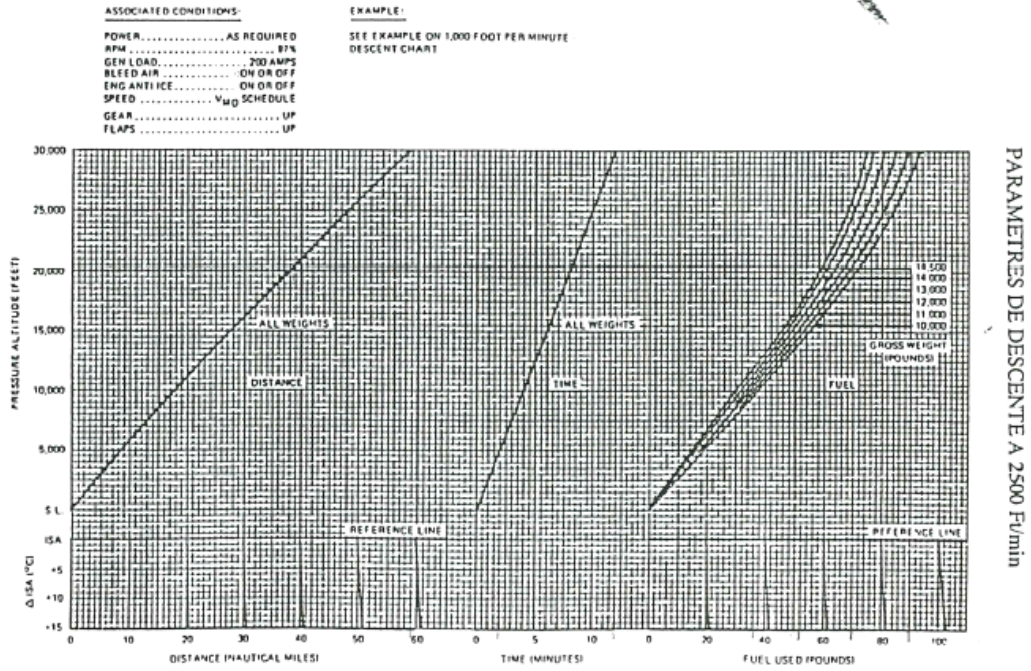


Figure 6-4: Distance, time and fuel in a descent phase

Page intentionally left blank

7. APPENDIX B

The appendix B provides operations performance parameters summary and airlines performance speed schedules summary with sources. The details about parameters may be found in [1].

Table 7-1: Operations performance parameters summary table

| Model Category | Symbols | Units | Source |
|---|--|---|--|
| Aircraft type (3 values) | n_{eng} engine type wake category | dimensionless string string | reference data reference data ICAO data |
| Mass (4 values) | m_{ref} m_{min} m_{max} m_{pyld} | tonnes tonnes tonnes tonnes | computed value reference data reference data reference data |
| Flight envelope (6 values) | V_{MO} M_{MO} h_{MO} h_{max} G_w G_t | knots (CAS) dimensionless feet feet feet/kg feet/C | reference data reference data identified value reference data identified value identified value |
| Aerodynamics (13 values) (15 values for jet aircraft) | S $C_{D0,CR}$ $C_{D2,CR}$ $C_{D0,AP}$ $C_{D2,AP}$ $C_{D0,LD}$ $C_{D2,LD}$ $C_{D0,ALDG}$ $(V_{stall})_{ALL}$ $C_{Lbo(M=0)}$ K | m^2 dimensionless dimensionless dimensionless dimensionless dimensionless dimensionless dimensionless knots (CAS, TO, IC, CR, AP, LD) dimensionless [1/M] | reference data identified data identified data referent data referent data referent data referent data referent data referent data identified data identified data |

| Model Category | Symbols | Units | Source |
|-------------------------------|---|--|---|
| Engine thrust (12 values) | $C_{Tc,1}$ $C_{Tc,2}$ $C_{Tc,3}$ $C_{Tc,4}$ $C_{Tc,5}$ $C_{Tdes,low}$ $C_{Tdes,high}$ h_{des} $C_{Tdes,app}$ $C_{Tdes,ld}$ $V_{des,ref}$ $M_{des,ref}$ | Newton (jet/piston) knot-Newton (turboprop) feet 1/feet ² (jet) Newton (turboprop) knot-Newton (piston) deg. C 1/ deg. C dimensionless dimensionless feet dimensionless dimensionless knots dimensionless | identified data identified data identified data identified data identified data identified data identified data identified data identified data reference data reference data |
| fuel flow (5 values) | C_{f1} C_{f2} C_{f3} C_{f4} C_{fcr} | kg/(min*kN) (jet) kg/(min*kN*knot) (turboprop) kg/min (piston) knots kg/min feet dimensionless | identified data identified data identified data identified data identified data |
| Ground movement (4 values) | <i>TOL</i> <i>LDL</i> <i>span</i> <i>length</i> | m m m m | reference data reference data reference data reference data |

Table 7-2: Airlines performance speed schedules summary table

| Profile | Symbol | Unit | Source |
|---------|--------|-----------------------|----------------|
| Climb | CAS lo | knots | computed value |
| | CAS hi | knots | computed value |
| | Mach | dimensionless (100 *) | computed value |
| Cruise | CAS lo | knots | computed value |
| | CAS hi | knots | computed value |
| | Mach | dimensionless (100 *) | computed value |
| Descent | Mach | dimensionless (100 *) | computed value |
| | CAS hi | knots | computed value |
| | CAS lo | knots | computed value |

Short description of source values used in above tables:

- reference data – data taken from aircraft performance reference data;
- identified data – data identified in the identification process;
- ICAO data – data taken from ICAO publications;
- computed data – data computed in the data processing phase.

Page intentionally left blank

8. REFERENCES

- [1] User manual for the base of aircraft data (BADA) – Revision 3.7, EEC Technical Report No. 2009-003, EUROCONTROL EEC, 2009.
- [2] Doc 8643 - Edition 36, ICAO Web site, <http://www.icao.int/anb/ais/8643/index.cfm>, last updated: 17. November 2008.
- [3] Model Accuracy Summary Report for the Base of Aircraft Data (BADA) – Revision 3.7, EEC Technical Report No. 2009-006, EUROCONTROL EEC, 2009.

Page intentionally left blank